
Contents

I Convex Optimization over Symmetric Cone	1
1 Cones, Complementarity, and Conic Optimization	3
1.1 Proper Cones and Conic Inequalities	3
1.1.1 Convex sets and cones	3
1.1.2 Partial order induced by proper cone	5
1.2 Complementarity over Cones	6
1.2.1 Dual cones and self-duality	6
1.2.2 Complementarity problems	7
1.2.3 Variational inequalities	8
1.2.4 Complementarity over nonnegative orthant	9
1.2.5 Overview of complementarity over cones	10
1.3 Positive-Semidefinite Cone	11
1.3.1 Positive-semidefinite matrices	12
1.3.2 Inner product of matrices	16
1.3.3 Self-duality of positive-semidefinite cone	17
1.3.4 Complementarity over positive-semidefinite cone	18
1.4 Second-Order Cone	19
1.4.1 Fundamentals of second-order cone	20
1.4.2 Self-duality of second-order cone	20
1.4.3 Complementarity over second-order cone	22
1.5 Conic Constraints and Their Relationship	26
1.6 Conic Optimization	29
1.6.1 Linear programming	30
1.6.2 Semidefinite programming	32
1.6.3 Second-order cone programming	33
1.7 Notes	36
2 Optimality and Duality	39
2.1 Fundamentals of Convex Analysis	39
2.1.1 Convex sets and convex functions	40
2.1.2 Monotone functions and convexity	41
2.1.3 Closed convex functions	44
2.1.4 Subdifferential	45
2.1.5 Conjugate function	47
2.2 Optimality and Duality	50
2.2.1 Dual problem	50

2.2.2	Weak duality	51
2.2.3	Strong duality	53
2.2.4	Optimality condition	54
2.2.5	Fenchel duality	55
2.2.6	Lagrangian duality	58
2.2.7	KKT conditions	61
2.3	Application to Semidefinite Programming	63
2.3.1	Fenchel dual problem of SDP	63
2.3.2	Duality and optimality of SDP	66
2.3.3	Lagrangian duality of SDP	69
2.4	Notes	72
3	Applications in Structural Engineering	73
3.1	Compliance Optimization	73
3.1.1	Definition of compliance	74
3.1.2	Compliance minimization	76
3.1.3	Worst-case compliance and robust optimization	79
3.2	Eigenvalue Optimization	81
3.2.1	Eigenvalue optimization of structures	81
3.2.2	SDP formulation	82
3.2.3	Optimality condition	84
3.3	Set-Valued Constitutive Law	86
3.3.1	Constitutive law	86
3.3.2	Linear elasticity and Legendre transformation	88
3.3.3	Inversion via Fenchel transformation	89
3.3.4	Unilateral contact law and Fenchel transformation	91
3.4	Notes	94
II	Cable Networks: An Example in Nonsmooth Mechanics	97
4	Principles of Potential Energy for Cable Networks	99
4.1	Constitutive law	99
4.1.1	No-compression model	100
4.1.2	Inclusion form	101
4.1.3	Variational form	103
4.1.4	Complementarity form	104
4.2	Potential Energy Principles in Convex Optimization Forms .	108
4.2.1	Principle of potential energy in general form	108
4.2.2	Principle for large strain	112
4.2.3	Principle for linear strain	116
4.2.4	Principle for the Green–Lagrange strain	117
4.3	More on Cable Networks: Nonlinear Material Law	119
4.3.1	Piecewise-linear law	120
4.3.2	Piecewise-quadratic law	124

4.4 Notes	127
5 Duality in Cable Networks: Principles of Complementary Energy	129
5.1 Duality in Cable Networks (1): Large Strain	130
5.1.1 Embedding to Fenchel form	130
5.1.2 Dual problem	131
5.1.3 Duality and optimality	135
5.1.4 Principle of complementary energy	139
5.1.5 Existence and uniqueness of solution	145
5.2 Duality in Cable Networks (2): Linear Strain	147
5.2.1 Embedding to Fenchel form	148
5.2.2 Dual problem	149
5.2.3 Duality and optimality	150
5.2.4 Principle of complementary energy	152
5.3 Duality in Cable Networks (3): Green–Lagrange Strain	153
5.3.1 Embedding to Fenchel form	153
5.3.2 Dual problem	155
5.3.3 Duality and optimality	157
5.3.4 Principle of complementary energy	161
5.4 Notes	163
III Numerical Methods	165
6 Algorithms for Conic Optimization	167
6.1 Primal-Dual Interior-Point Method	167
6.1.1 Outline of interior-point methods	167
6.1.2 Interior-point method for linear programming	168
6.1.3 Interior-point method for semidefinite programming .	173
6.2 Reformulation and Smoothing Method	177
6.2.1 Reformulation method	177
6.2.2 Smoothing method	180
6.2.3 Extensions to conic complementarity problems	181
6.3 Notes	183
7 Numerical Analysis of Cable Networks	185
7.1 Cable Networks with Pin-Joints	185
7.2 Cable Networks with Sliding Joints	195
7.3 Form-Finding of Cable Networks	200
7.3.1 Form-finding with specified axial forces	201
7.3.2 Special cases	202
7.4 Notes	206
IV Problems in Nonsmooth Mechanics	209

8 Masonry Structures	211
8.1 Introduction	211
8.1.1 Notation	213
8.2 Principle of Potential Energy for Masonry Structures	214
8.2.1 Principle of potential energy	214
8.2.2 Constitutive law	216
8.2.3 Conic optimization formulation	221
8.3 Principle of Complementary Energy for Masonry Structures	225
8.3.1 Embedding to Fenchel form	225
8.3.2 Dual problem	228
8.3.3 Duality and optimality	232
8.3.4 Principle of complementary energy	235
8.4 Numerical Aspects	237
8.4.1 Spatial discretization	237
8.4.2 Examples	243
8.5 Notes	249
9 Planar Membranes	253
9.1 Introduction	253
9.2 Analysis in Small Deformation	255
9.2.1 Principle of potential energy in small deformation	255
9.2.2 Conic optimization formulation	259
9.2.3 Principle of complementary energy in small deformation	261
9.3 Principle of Potential Energy for Membranes	264
9.3.1 Constitutive law	264
9.3.2 Principle of potential energy	273
9.4 Principle of Complementary Energy for Membranes	274
9.4.1 Embedding to Fenchel form	275
9.4.2 Dual problem	276
9.4.3 Duality and optimality	280
9.4.4 Principle of complementary energy	288
9.5 Numerical Aspects	291
9.5.1 Spatial discretization	291
9.5.2 Examples	295
9.6 Notes	305
10 Frictional Contact Problems	311
10.1 Friction Law	311
10.1.1 Coulomb's law	312
10.1.2 Second-order cone complementarity formulation	314
10.2 Incremental Problem	317
10.2.1 Friction law in incremental problems	318
10.2.2 Contact kinematics	318
10.2.3 Problem formulation	321
10.3 Discussions on Various Complementarity Forms	329

<i>Contents</i>	xix
10.3.1 On auxiliary variables	329
10.3.2 Maximum dissipation law and its optimality conditions	330
10.3.3 A formulation using projection operator	339
10.3.4 Friction law and normality rule	340
10.4 Notes	348
11 Plasticity	351
11.1 Fundamentals of Plasticity	351
11.2 Perfect Plasticity	356
11.2.1 Classical formulation of flow rule in perfect plasticity .	356
11.2.2 Second-order cone complementarity formulation	358
11.3 Plasticity with Isotropic Hardening	362
11.3.1 Linear isotropic hardening law	363
11.3.2 Second-order cone complementarity formulation	364
11.3.3 Incremental problem	367
11.3.4 SOCP formulation of incremental problem	370
11.4 Plasticity with Kinematic Hardening	373
11.4.1 Linear kinematic hardening	374
11.4.2 Second-order cone complementarity formulation	375
11.4.3 SOCP formulation of incremental problem	377
11.5 Notes	379
References	381
Index	417
About the Author	425
