Discrete Optimization of Damper Placement in a Shear Building via Mixed Integer Programming

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optimal placement of supplemental dampers

- linear-quadratic regulator (optimal control)

  [Gluck, Reinhorn, Gluck & Levy '96]

- sequential search algorithm [Shukla & Datta '99] [López García '01]

  (heuristics introducing damper units sequentially)

- minimization of transfer function

  [Takewaki '97] [Takewaki & Yoshitomi '98] [Cimellaro '07] [Aydin '12]

- analogy of fully-stressed design [Lavan & Levy '06]

optimal placement of supplemental dampers

- linear-quadratic regulator (optimal control)

  [Gluck, Reinhorn, Gluck & Levy ’96]

- sequential search algorithm [Shukla & Datta ’99] [López García ’01$(*1)$]

  (heuristics introducing damper units sequentially)

- minimization of transfer function

  [Takewaki ’97$(*2)$] [Takewaki & Yoshitomi ’98] [Cimellaro ’07] [Aydin ’12]

- analogy of fully-stressed design [Lavan & Levy ’06$(*3)$]

- comparison of $(*1)$, $(*2)$, & $(*3)$

  [Whittle, Williams, Karavasilis & Blakeborough ’12]

  - broadly comparable performances

- this study: “$(*2)$ & discrete variables” $\leftarrow$ global optimization

  (damping coefficient) $\in \{0, \bar{c}, 2\bar{c}, 3\bar{c}, \dots\}$

mixed-integer programming

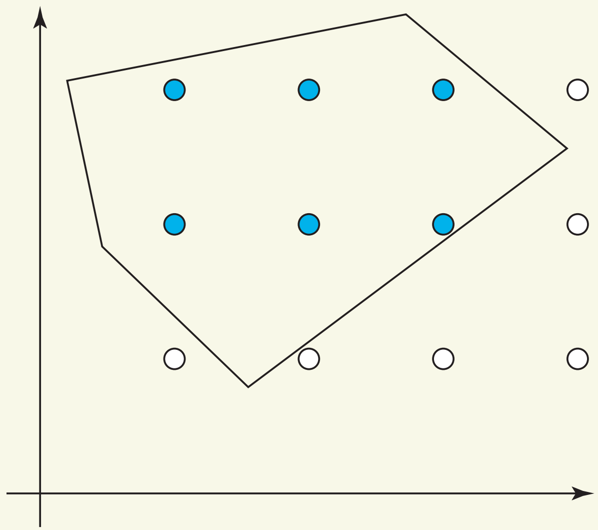
- m-i linear prog.:

$$\begin{aligned} \min \quad & \boldsymbol{f}^{\mathrm{T}}\boldsymbol{x} + \boldsymbol{r}^{\mathrm{T}}\boldsymbol{y} \\ \text{s.t.} \quad & A\boldsymbol{x} + G\boldsymbol{y} \leq \boldsymbol{b} \\ & \boldsymbol{x} \in \{0,1\}^n, \quad \boldsymbol{y} \in \mathbb{R}^m \end{aligned}$$

mixed-integer programming

- m-i linear prog.:

$$\begin{array}{ll} \min & \boldsymbol{f}^{\mathrm{T}}\boldsymbol{x} + \boldsymbol{r}^{\mathrm{T}}\boldsymbol{y} \\ \mathrm{s.\,t.} & A\boldsymbol{x} + G\boldsymbol{y} \leq \boldsymbol{b} \\ & \boldsymbol{x} \in \{0, 1\}^n, \quad \boldsymbol{y} \in \mathbb{R}^m \end{array}$$

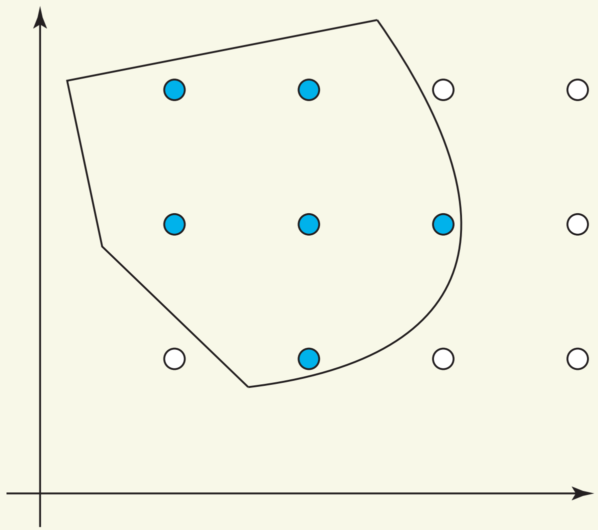


- replace $\boldsymbol{x} \in \{0, 1\}^n$ with $\mathbf{0} \leq \boldsymbol{x} \leq \mathbf{1}$ $\quad\rightarrow$ linear prog.
- can be solved with, e.g., branch-and-bound method

mixed-integer programming

- m-i second-order cone prog.:

$$\begin{array}{ll} \min & \boldsymbol{f}^{\mathrm{T}}\boldsymbol{x} + \boldsymbol{r}^{\mathrm{T}}\boldsymbol{y} \\ \mathrm{s.\,t.} & \|A_l\boldsymbol{x} + G_l\boldsymbol{y} - \boldsymbol{b}_l\| \leq \boldsymbol{d}_l^{\mathrm{T}}\boldsymbol{x} + \boldsymbol{e}_l^{\mathrm{T}}\boldsymbol{y} - h_l \\ & \boldsymbol{x} \in \{0,1\}^n, \quad \boldsymbol{y} \in \mathbb{R}^m \end{array}$$

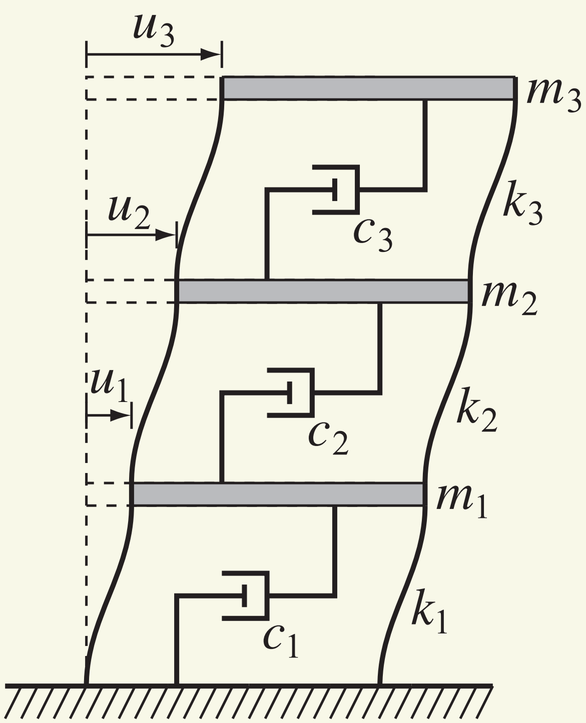


- replace $\boldsymbol{x} \in \{0,1\}^n$ with $\mathbf{0} \leq \boldsymbol{x} \leq \mathbf{1}$ $\rightarrow$ convex prog. (s-o cone prog.)
- can be solved with, e.g., branch-and-bound method

eq. of motion (n-story shear building model)

$$\boldsymbol{K}\boldsymbol{u} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{M}\ddot{\boldsymbol{u}} = -\boldsymbol{M}\ddot{u}_{\mathrm{g}}\mathbf{1}$$

- $\boldsymbol{u} \in \mathbb{R}^n$: floor disp. vector
- $\ddot{u}_{\mathrm{g}}$: base acceleration

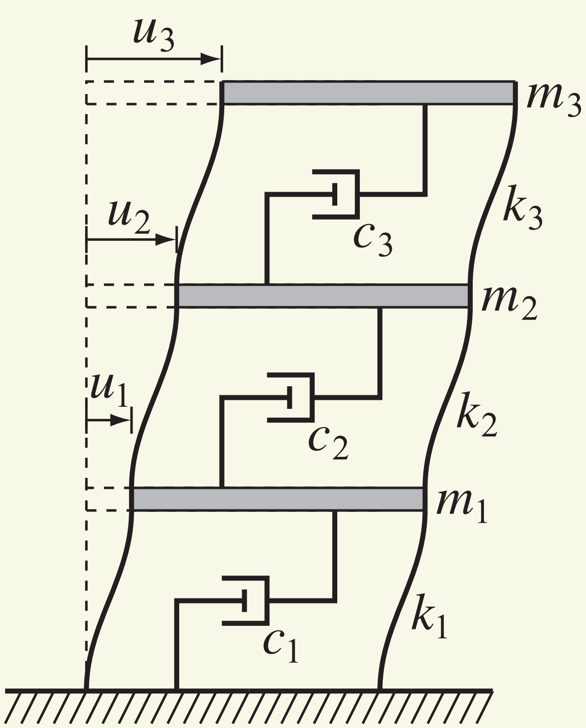


$n = 3$ story model

eq. of motion (n-story shear building model)

$$K\boldsymbol{u} + C\dot{\boldsymbol{u}} + M\ddot{\boldsymbol{u}} = -M\ddot{u}_{\mathrm{g}}\mathbf{1}$$

- $\boldsymbol{u} \in \mathbb{R}^n$: floor disp. vector
- $\ddot{u}_{\mathrm{g}}$: base acceleration
- K : stiffness matrix
- $C(\boldsymbol{c})$: damping matrix
  - c_i : damping coefficient of damper i
    ← design variable
- M : mass matrix



$n = 3$ story model

transfer fcn. of interstory drift

- eq. of motion in freq. domain:

$$(K + \mathrm{i}\omega C - \omega^2 M)\boldsymbol{v}(\omega) = -M\ddot{v}_{\mathrm{g}}(\omega)\mathbf{1}$$

  - $\boldsymbol{v}(\omega)$: Fourier transform of floor disp. $\boldsymbol{u}$
  - $\ddot{v}_{\mathrm{g}}(\omega)$: Fourier transform of base accel. $\ddot{u}_{\mathrm{g}}$

- transfer fcn. of $\boldsymbol{u}$: ($\bar{\omega}$: fundamental freq.)

$$\hat{\boldsymbol{v}} = \boldsymbol{v}(\bar{\omega})/\ddot{v}_{\mathrm{g}}(\bar{\omega})$$

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$$\hat{\boldsymbol{v}} = \boldsymbol{v}(\bar{\omega})/\ddot{v}_{\mathrm{g}}(\bar{\omega})$$

- transfer fcn. of interstory drifts:

$$\hat{\boldsymbol{\delta}} = H^{\mathrm{T}}\hat{\boldsymbol{v}}$$

  - interstory drifts: $\boldsymbol{d} = H^{\mathrm{T}}\boldsymbol{u}$ with constant matrix H
  - [Takewaki '97] : Minimize $\sum_{i=1}^{n} |\hat{\delta}_i|$

optimal placement problem [Takewaki '97]

- min. of sum of transfer fcn. ampls. of interstory drifts:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{n} |\hat{\delta}_i| \\
\text{s.t.} \quad & \hat{\boldsymbol{\delta}} = H^{\mathrm{T}} \hat{\boldsymbol{v}} && \text{(def. of } \hat{\delta}_i\text{)} \\
& (K + \mathrm{i}\bar{\omega} C(\boldsymbol{c}) - \bar{\omega}^2 M)\hat{\boldsymbol{v}} = -M\mathbf{1} && \text{(eq. of m.)} \\
& \sum_{i=1}^{n} c_i \le c_{\mathrm{sum}}^{\max} && \text{(upper bound)} \\
& c_i \ge 0 \quad (i = 1, \ldots, n)
\end{aligned}
$$

- variables are
  - $\boldsymbol{c} \in \mathbb{R}^n$: damper damping coeffs.
  - $\hat{\boldsymbol{v}} \in \mathbb{C}^n$: transfer fcn. of floor disps.
  - $\hat{\boldsymbol{\delta}} \in \mathbb{C}^n$: transfer fcn. of interstory drifts

discrete optimization

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{n} |\hat{\delta}_i| \\
\text{s.t.} \quad & \hat{\boldsymbol{\delta}} = H^{\mathrm{T}} \hat{\boldsymbol{v}} \\
& (K + \mathrm{i}\bar{\omega} C(\boldsymbol{c}) - \bar{\omega}^2 M)\hat{\boldsymbol{v}} = -M\mathbf{1} \\
& \sum_{i=1}^{n} c_i \leq c_{\mathrm{sum}}^{\mathrm{max}} \\
& c_i \in \{0, \bar{c}, 2\bar{c}, \ldots, p\bar{c}\} \quad (i = 1, \ldots, n) \qquad (\clubsuit)
\end{aligned}
$$

- ($\clubsuit$) : choose c_i among available candidates
  - "0" or "a multiple of $\bar{c}$"
- $\longrightarrow$ propose a global opt. approach
  - mixed-integer programming

towards mixed-integer programming

- use of 0–1 variables:

$$x_{ij} \in \{0, 1\}, \quad x_{i1} \geq x_{i2} \geq \cdots \geq x_{ip}$$

  - choice of damper c_i :

$$c_i \in \{0, \bar{c}, 2\bar{c}, \ldots, p\bar{c}\} \quad \Leftrightarrow \quad c_i = \bar{c} \sum_{j=1}^{p} x_{ij}$$

towards mixed-integer programming

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- min. of $\sum_{i=1}^{n} |\hat{\delta}_i|$: $(\hat{\delta}_i \in \mathbb{C})$

$$\min \sum_{i=1}^{n} y_i \quad \text{s. t. } y_i \geq \|(\operatorname{Re} \delta_i, \operatorname{Im} \delta_i)\| \qquad \text{(soc)}$$

- min. of $\max\{|\hat{\delta}_1|, \ldots, |\hat{\delta}_6|\}$:

$$\min y \quad \text{s. t. } y \geq \|(\operatorname{Re} \delta_i, \operatorname{Im} \delta_i)\| \qquad \text{(soc)}$$

mixed-integer second-order cone programming formulation

$$\min \quad \sum_{i=1}^{n} y_i$$

$$\text{s.t.} \quad y_i \geq \|(\mathrm{Re}\,\delta_i, \mathrm{Im}\,\delta_i)\|, \quad \hat{\boldsymbol{\delta}} = H^{\mathrm{T}} \hat{\boldsymbol{v}}$$

$$(K - \bar{\omega}^2 M)\hat{\boldsymbol{v}} + \mathrm{i}\bar{\omega} H \boldsymbol{q} = -M\mathbf{1} \qquad \text{(eq. of m.)}$$

$$q_i = \bar{c} \sum_{j=1}^{p} w_{ij} \qquad \text{(t. fcn. of story shear)}$$

$$|w_{ij}| \leq \mu x_{ij}, \quad |w_{ij} - \boldsymbol{h}_i^{\mathrm{T}} \hat{\boldsymbol{v}}| \leq \mu(1 - x_{ij})$$

$$x_{ij} \in \{0, 1\}, \quad x_{i1} \geq x_{i2} \geq \cdots \geq x_{ip}$$

(μ : large constant)

- replace $x_{ij} \in \{0, 1\}$ with $0 \leq x_{ij} \leq 1$ → convex prog. (SOCP)
- can be solved by a branch-and-bound method
- software packages are available (e.g., Gurobi Optimizer, CPLEX)

more constraints on damper damping coefficients

- 0–1 variables (as before):

$$x_{ij} \in \{0, 1\}, \quad x_{i1} \geq x_{i2} \geq \cdots \geq x_{ip}$$

  - choice of damper c_i :

$$c_i \in \{0, \bar{c}, 2\bar{c}, \ldots, p\bar{c}\} \quad \Leftrightarrow \quad c_i = \bar{c} \sum_{j=1}^{p} x_{ij}$$

- γ : upper bound for the # of damped stories

$$\sum_{i=1}^{n} x_{i1} \leq \gamma$$

more constraints on damper damping coefficients

- 0–1 variables (as before):

$$x_{ij} \in \{0, 1\}, \quad x_{i1} \geq x_{i2} \geq \cdots \geq x_{ip}$$

  - choice of damper c_i :

$$c_i \in \{0, \bar{c}, 2\bar{c}, \ldots, p\bar{c}\} \quad \Leftrightarrow \quad c_i = \bar{c} \sum_{j=1}^{p} x_{ij}$$

- $\bar{r}\bar{c}$: lower bound for damping coefficient
  (i.e., $c_i \in \{0, \bar{r}\bar{c}, (\bar{r}+1)\bar{c}, (\bar{r}+2)\bar{c}, \ldots, p\bar{c}\}$)

$$x_{i1} \leq x_{i\bar{r}} \quad (i = 1, \ldots, n)$$

  - $x_{i1} = 1 \Rightarrow x_{i2} = \cdots = x_{i\bar{r}} = 1$
  - $x_{i\bar{r}} = 0 \Rightarrow x_{i1} = \cdots = x_{i,\bar{r}-1} = 0$

more constraints on damper damping coefficients

- 0–1 variables (as before):

$$x_{ij} \in \{0, 1\}, \quad x_{i1} \geq x_{i2} \geq \cdots \geq x_{ip}$$

  - choice of damper c_i :

$$c_i \in \{0, \bar{c}, 2\bar{c}, \ldots, p\bar{c}\} \quad \Leftrightarrow \quad c_i = \bar{c} \sum_{j=1}^{p} x_{ij}$$

- at most 1 damper can be added to 2 adjacent stories:

$$x_{i1} + x_{i+1,1} \leq 1 \quad (i = 1, \ldots, n-1)$$

- at most 1 damper can be added to 3 adjacent stories:

$$x_{i1} + x_{i+1,1} + x_{i+2,1} \leq 1 \quad (i = 1, \ldots, n-2)$$

ex. 1) uniform stiffness distribution

- n = 6-story model
- mass: $m_i = 80{,}000\,\mathrm{kg}$ $(i = 1, \ldots, 6)$
- stiffness: $k_i = 40{,}000\,\mathrm{kN/m}$ $(i = 1, \ldots, 6)$
- damper damping coefficients c_i
    - upr. bd. for sum of c_i's: $c_{\mathrm{sum}}^{\mathrm{max}} = 9{,}000\,\mathrm{kNs/m}$

$\leftarrow$ same setting as [Takewaki '97]

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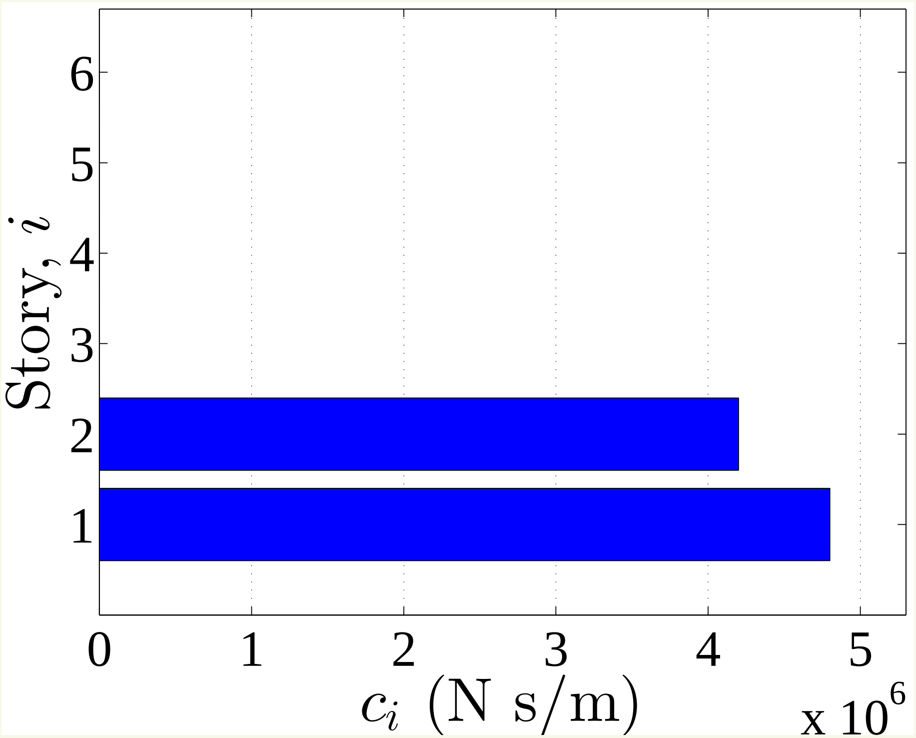
  - upr. bd. for sum of c_i's: $c_{\mathrm{sum}}^{\max} = 9{,}000\,\mathrm{kNs/m}$

    ← same setting as [Takewaki '97]
  - candidate values:
    - $c_i \in \{0, 500, 1000, \dots, 7500\}\,\mathrm{kNs/m}$
    - $c_i \in \{0, 200, 400, \dots, 6000\}\,\mathrm{kNs/m}$
    - $c_i \in \{0, 100, 200, \dots, 6000\}\,\mathrm{kNs/m}$

ex. 1) optimal solutions (uniform stiffness)

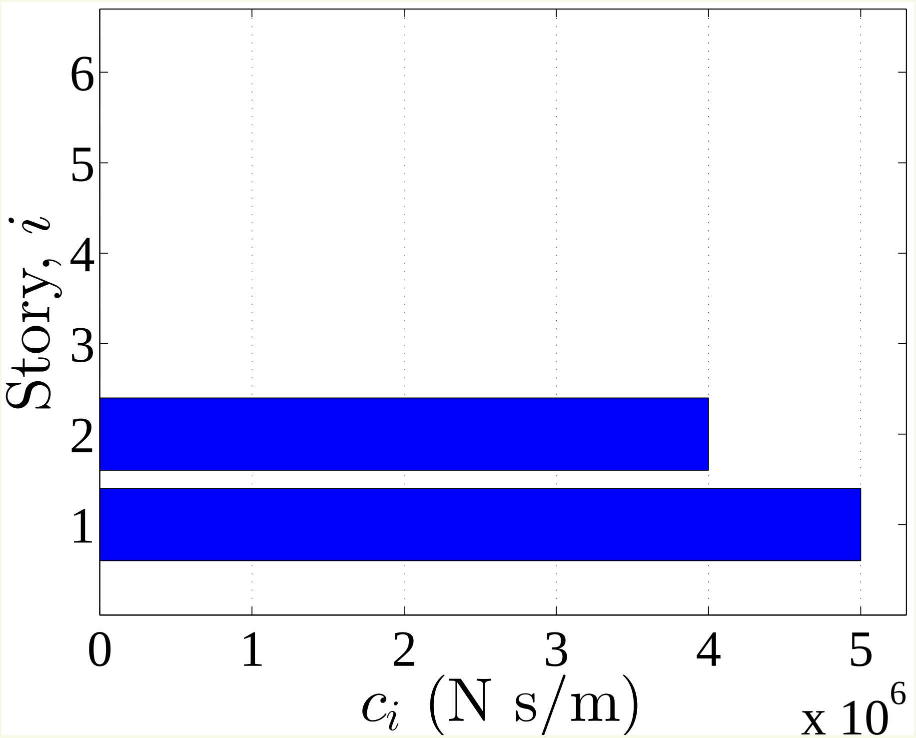
- damper damping coeffs.



$c_i \in \{0, 200, \ldots, 6000\}$ kNs/m

$c_i \in \{0, 100, \ldots, 6000\}$ kNs/m

$\sum_{i=1}^{6} |\hat{\delta}_i| = 0.135132$



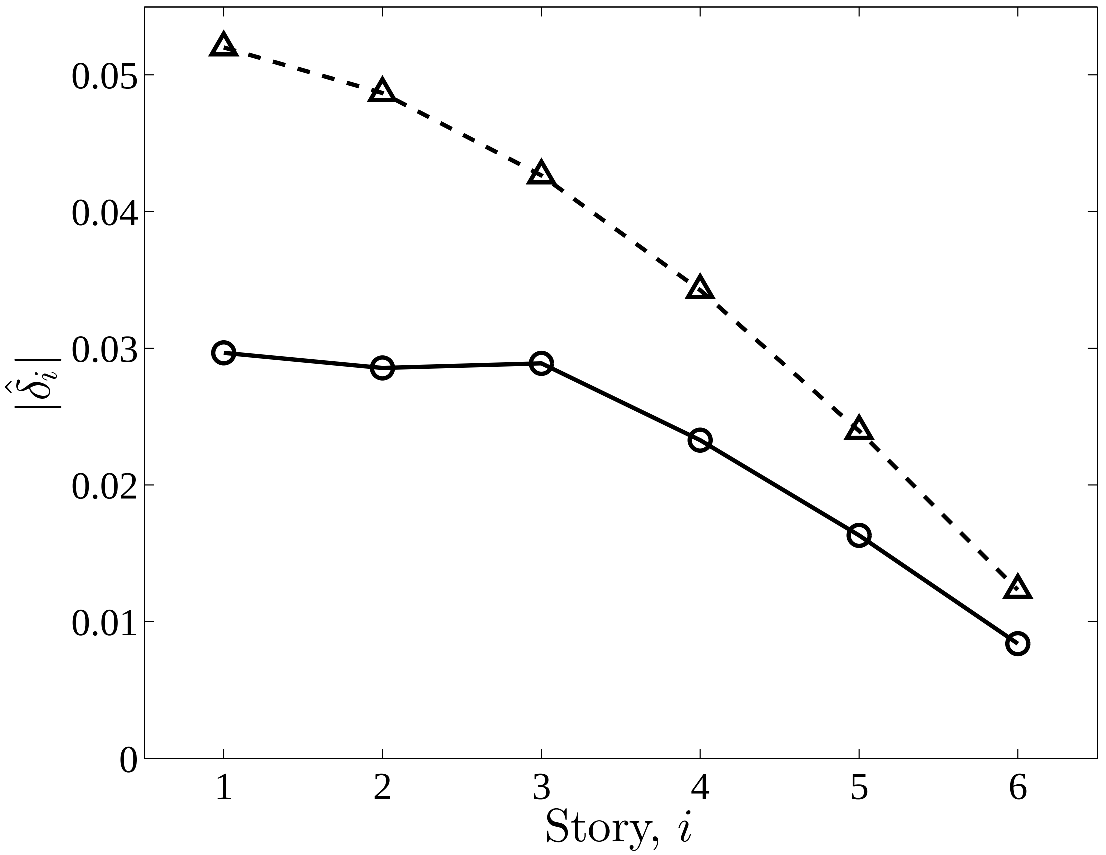
$c_i \in \{0, 500, \ldots, 7500\}$ kNs/m

$\sum_{i=1}^{6} |\hat{\delta}_i| = 0.1352236$

- similar to solutions in [Takewaki '97] (continuous opt.)

ex. 1) transfer functions (uniform stiffness)

- interstory drifts $|\hat{\delta}_i|$ (at fundamental freq. $\bar{\omega}$)



"———" optimal solution

"– – –" uniform damping $(c_1 = \cdots = c_6)$

- $|\hat{\delta}_i|$ is drastically decreased especially in the lower stories.

ex. 1) computational cost (uniform stiffness)

- comparison of two solvers

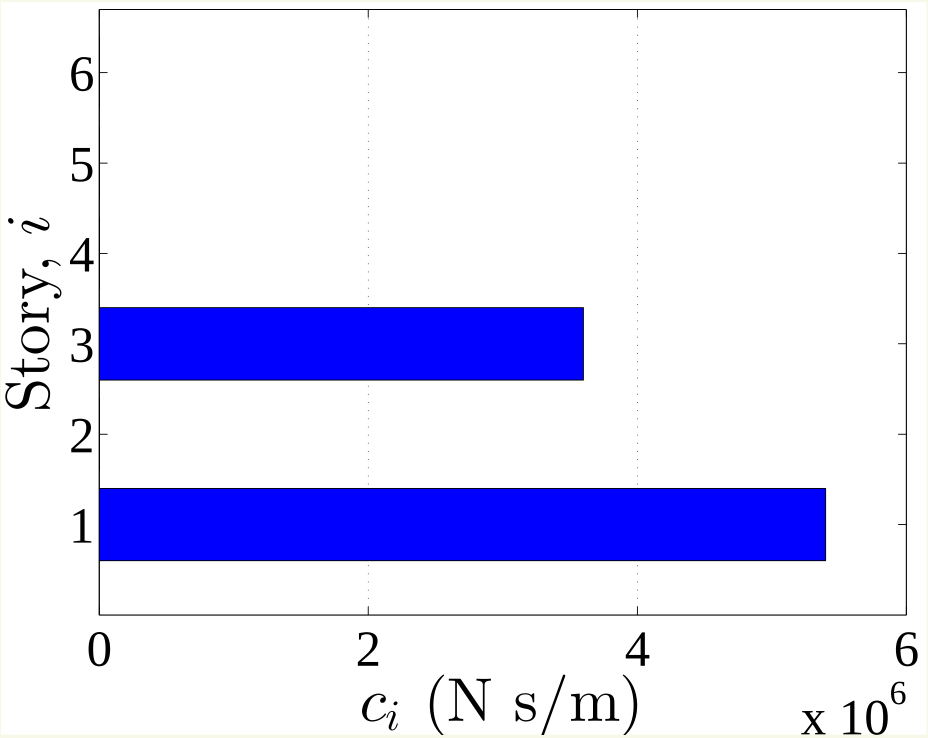
	CPLEX (ver. 12.2)		Gurobi (ver. 5.0)	
p	Time (s)	# of B&B nodes	Time (s)	# of B&B nodes
15	3.1	25,233	7.7	20,536
30	172.6	837,374	135.2	465,107
60	2,103.5	6,164,308	1,210.9	1,954,957

6-Core Intel Xeon Westmere (2.66 GHz) with 64 GB RAM

- problem size $(p = 60)$
  - # of 0–1 variables: 360
  - # of continuous variables: 402
  - # of lin. ineq. constraints: 1,615
  - # of lin. eq. constraints: 36
  - # of SOC constraints: 6

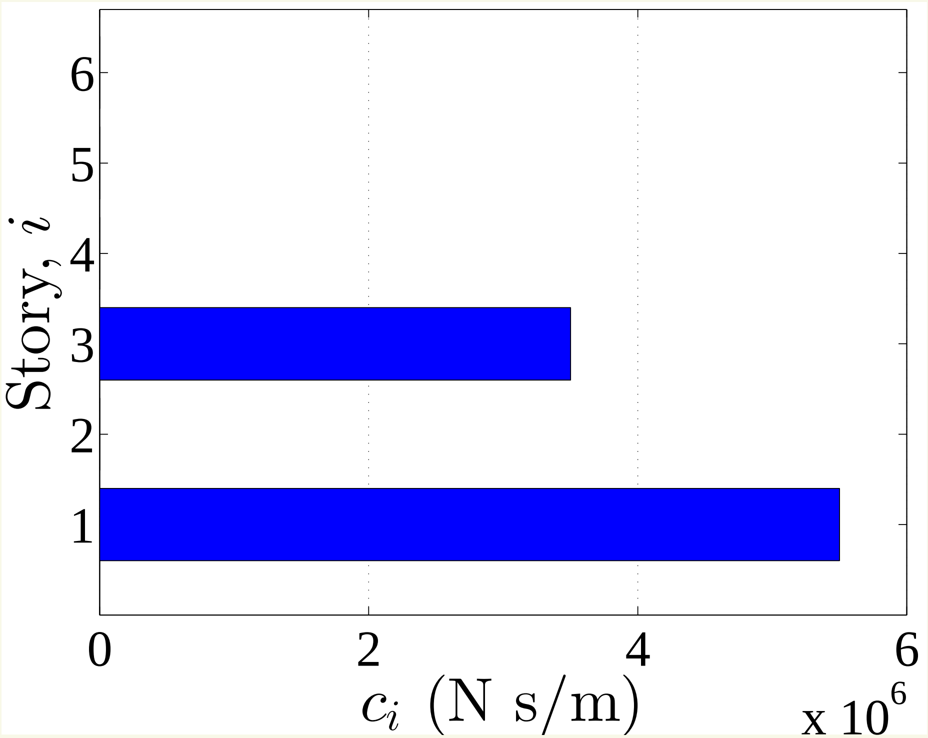
ex. 1) more constraints (uniform stiffness)

- constraints on damper damping coeffs.
  - upr. bd. for # of dampers: $\gamma = 3$
  - no adjacent damped stories



$c_i \in \{0, 200, \ldots, 6000\}$ kNs/m

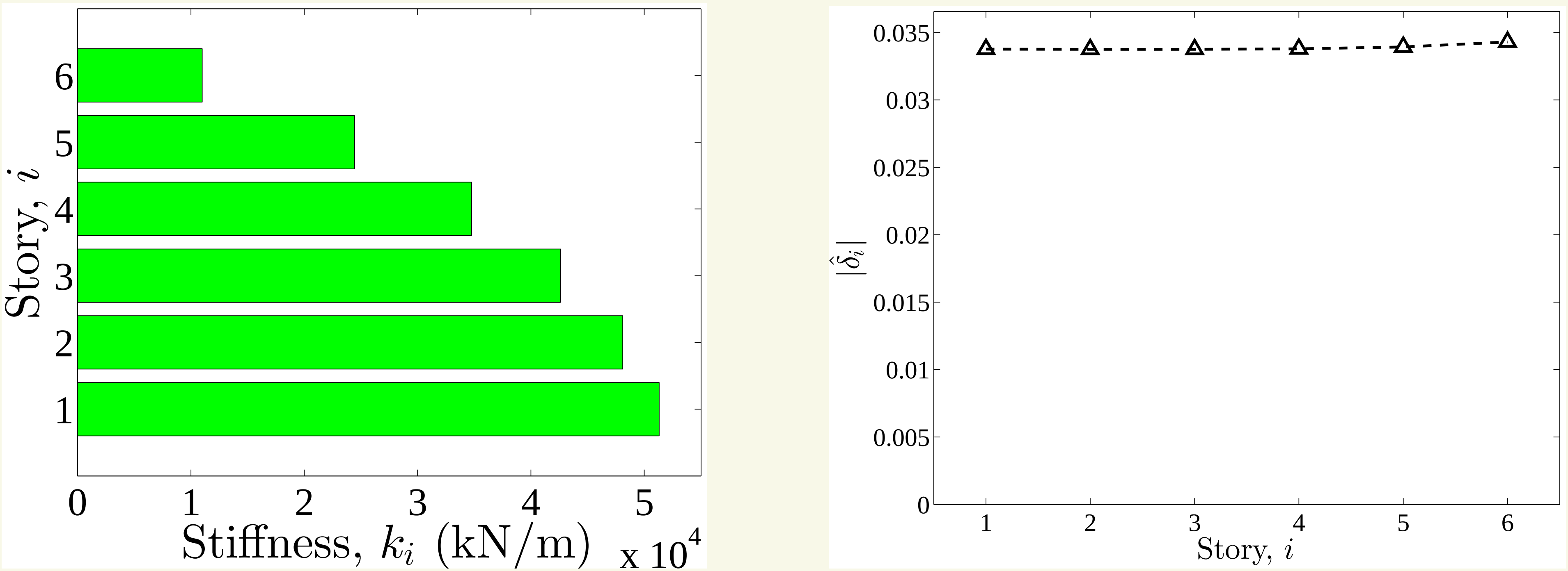
$c_i \in \{0, 100, \ldots, 6000\}$ kNs/m



$c_i \in \{0, 500, \ldots, 7500\}$ kNs/m

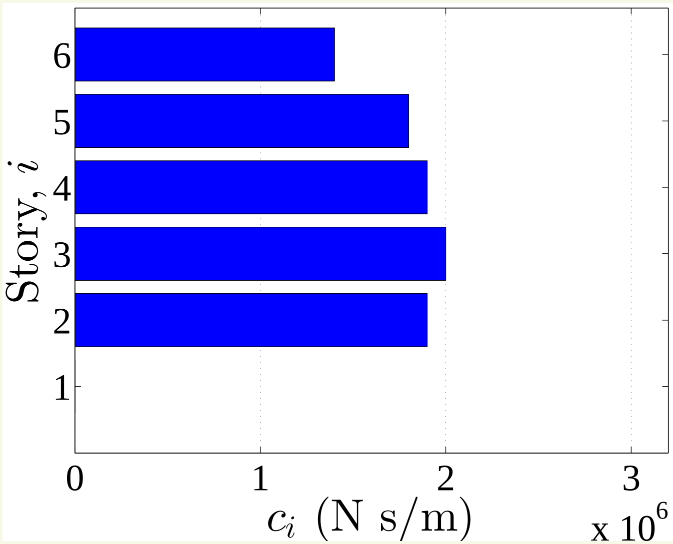
ex. 2) uniform distribution of amplitudes of transfer functions

- masses & damper damping coeffs.: same as the previous ex.
- distribution of story stiffnesses [Takewaki '97] :

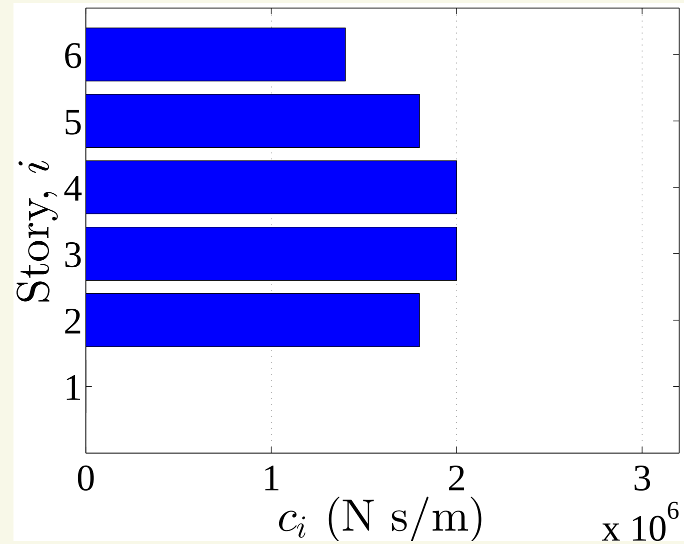


- With uniform damping $(c_1 = \cdots = c_6)$,
  distribution of $|\hat{\delta}_i|$ becomes uniform $(|\hat{\delta}_1| = \cdots = |\hat{\delta}_6|)$.

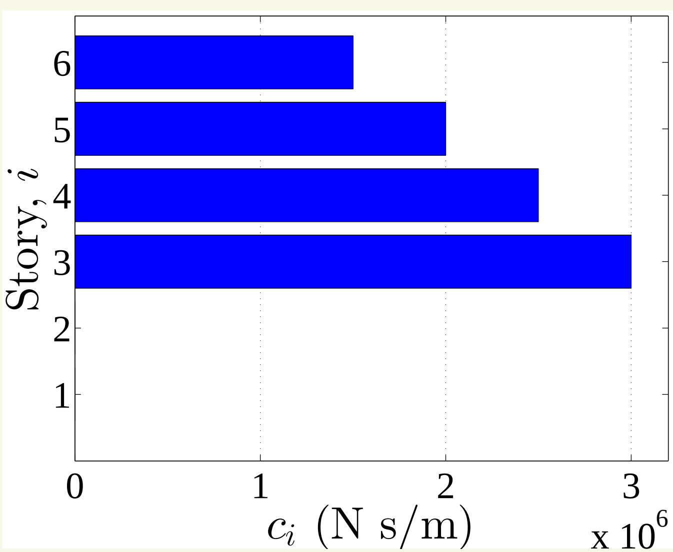
ex. 2) optimal solutions



(a) $c_i \in \{0, 100, \ldots, 6000\}$ kNs/m



(b) $c_i \in \{0, 200, \ldots, 6000\}$ kNs/m



(c) $c_i \in \{0, 500, \ldots, 7500\}$ kNs/m

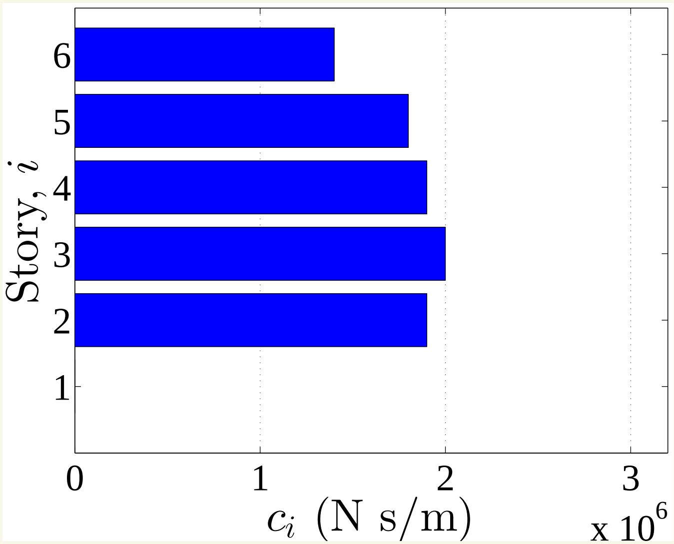
(a) obj. val. $= 0.201158$

(b) obj. val. $= 0.201162$

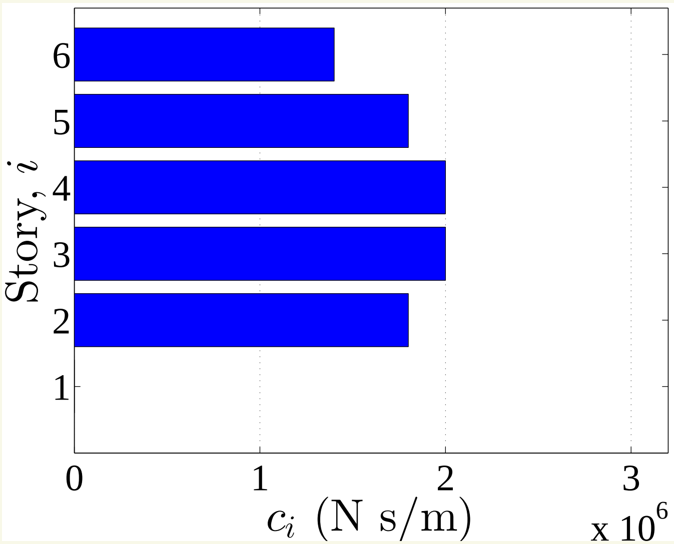
(c) obj. val. $= 0.201222$

- In [Takewaki '97] :
  obj. val. $= 0.2027$

ex. 2) optimal solutions



(a) $c_i \in \{0, 100, \ldots, 6000\}\,\mathrm{kNs/m}$

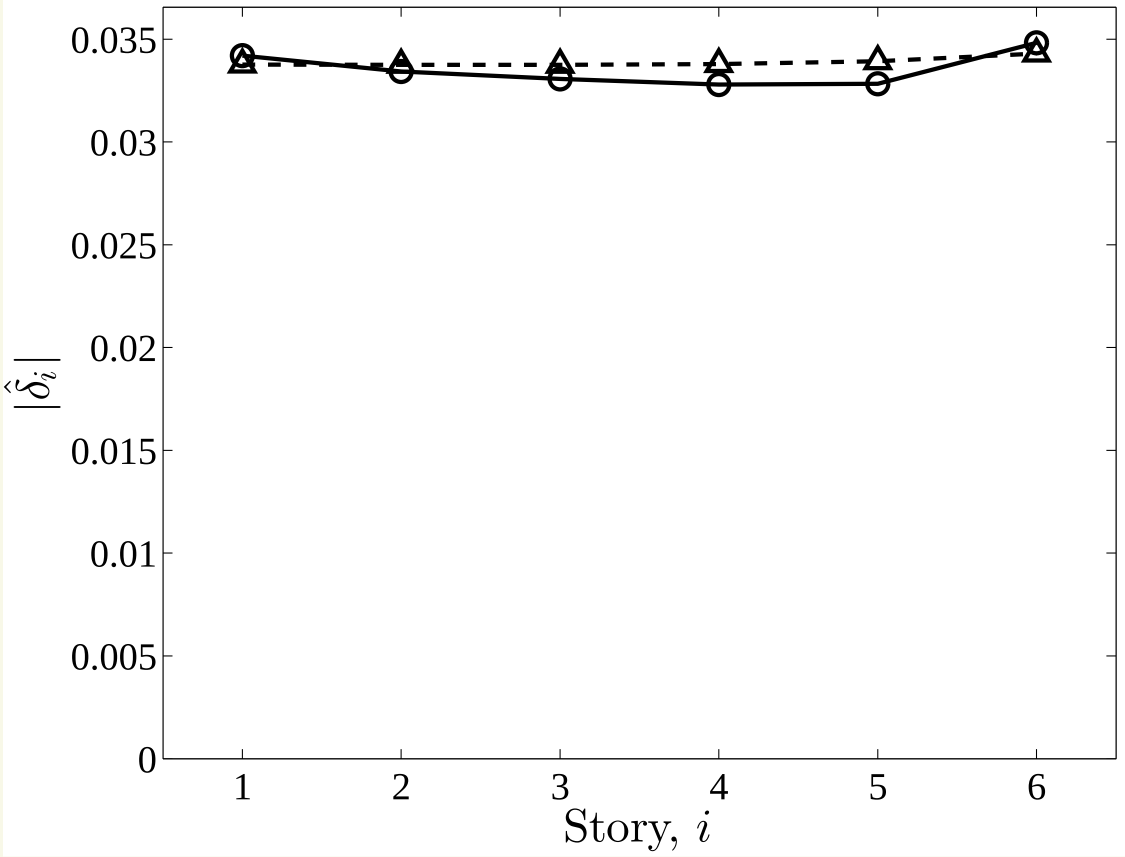


(b) $c_i \in \{0, 200, \ldots, 6000\}\,\mathrm{kNs/m}$

- [Takewaki '97] solved a continuous opt. prob.
  - There, dampers were placed to all stories.
- This study finds a global opt. sol. of a discrete prob.
  - The solution is better than the one in [Takewaki '97] .

ex. 2) transfer functions

- interstory drifts $|\hat{\delta}_i|$ (at fundamental freq. $\bar{\omega}$)



"——" optimal solution for $c_i \in \{0, 500, \ldots, 7500\}$ kNs/m
"– – –" uniform damping $(c_1 = \cdots = c_6 = 1{,}500\,\text{kNs/m})$

- $|\hat{\delta}_i|$'s are not decreased drastically.

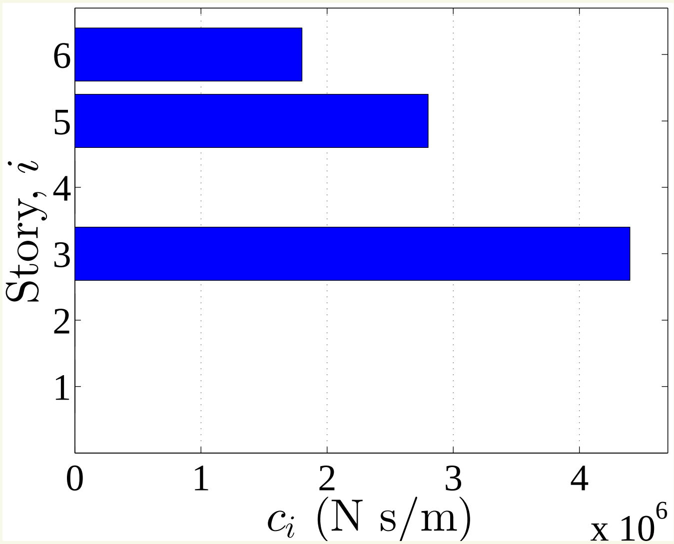
ex. 2) computational cost

	CPLEX (ver. 12.2)		Gurobi (ver. 5.0)	
p	Time (s)	# of B&B nodes	Time (s)	# of B&B nodes
15	26.3	146,817	16.6	100,999
30	1,455.6	6,158,001	880.6	4,623,129
60	62,021.6	128,500,335	33,917.6	88,934,141
	($\simeq$ 17.2 h)		($\simeq$ 9.4 h)	

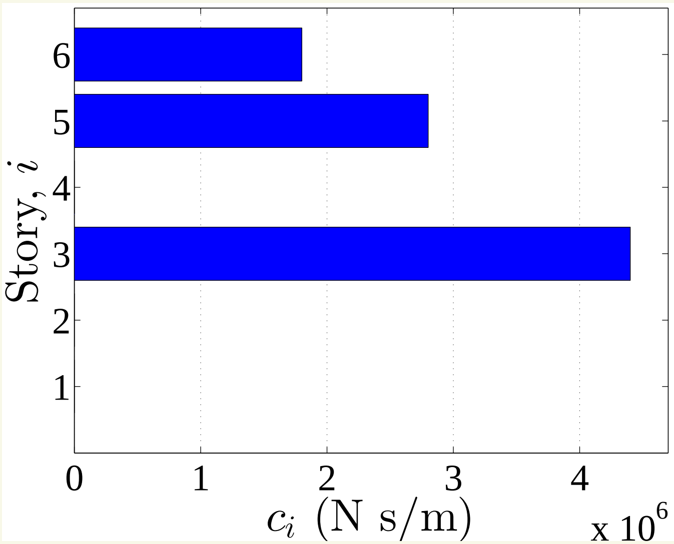
6-Core Intel Xeon Westmere (2.66 GHz) with 64 GB RAM

- probably a hard MIP problem
  - probably, very many “quite good” solutions & few “bad” solutions
    - uniform damping: obj. val. $= 0.203292$
    - optimal solution: obj. val. $= 0.201158$

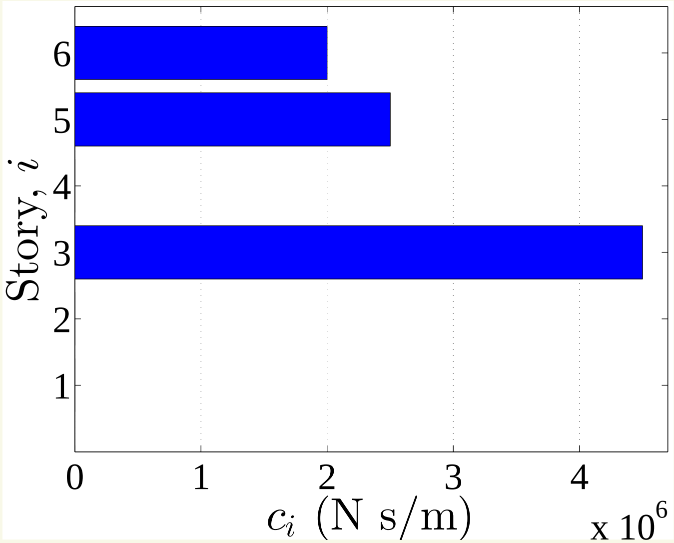
ex. 2) with more constraints



(a) $c_i \in \{0, 100, \ldots, 6000\}$ kNs/m



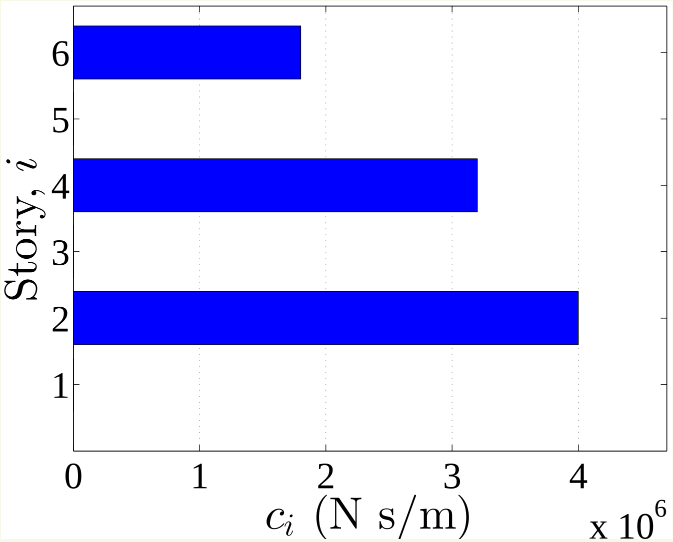
(b) $c_i \in \{0, 200, \ldots, 6000\}$ kNs/m



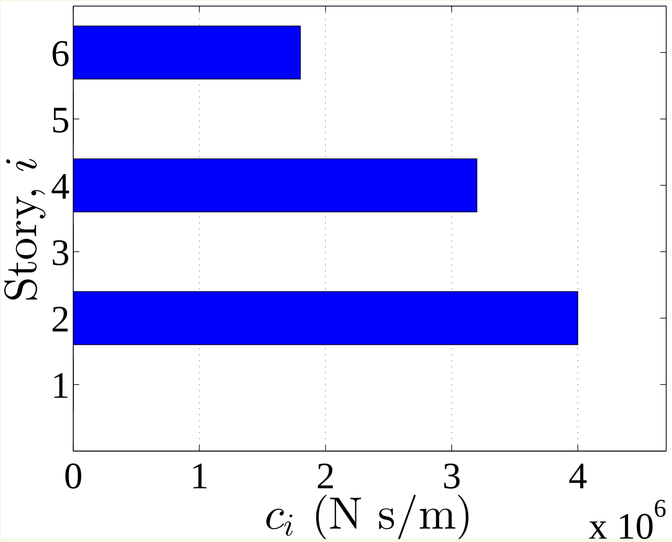
(c) $c_i \in \{0, 500, \ldots, 7500\}$ kNs/m

- upr. bd. for # of dampers: $\gamma = 3$

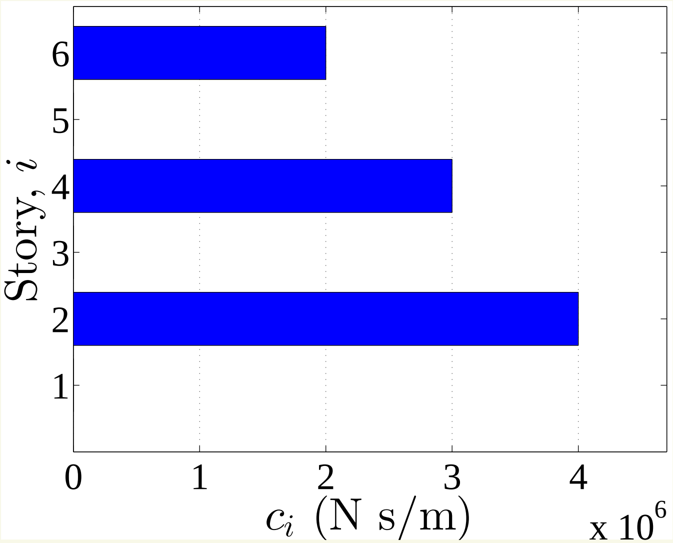
ex. 2) with more constraints



(a) $c_i \in \{0, 100, \ldots, 6000\}$ kNs/m



(b) $c_i \in \{0, 200, \ldots, 6000\}$ kNs/m



(c) $c_i \in \{0, 500, \ldots, 7500\}$ kNs/m

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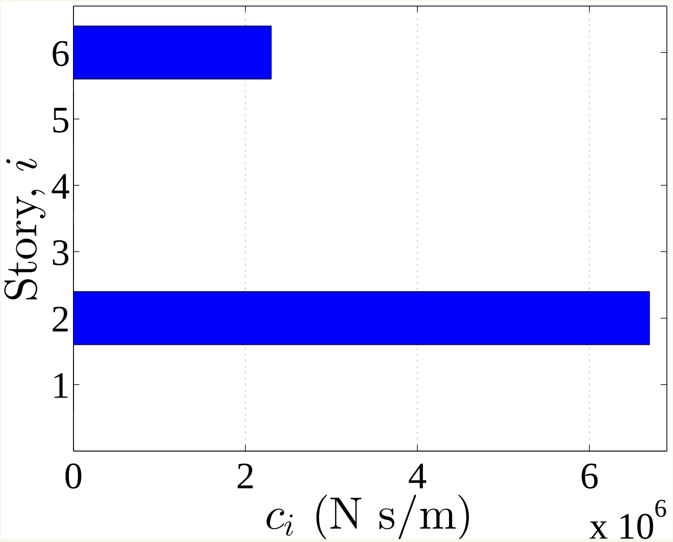
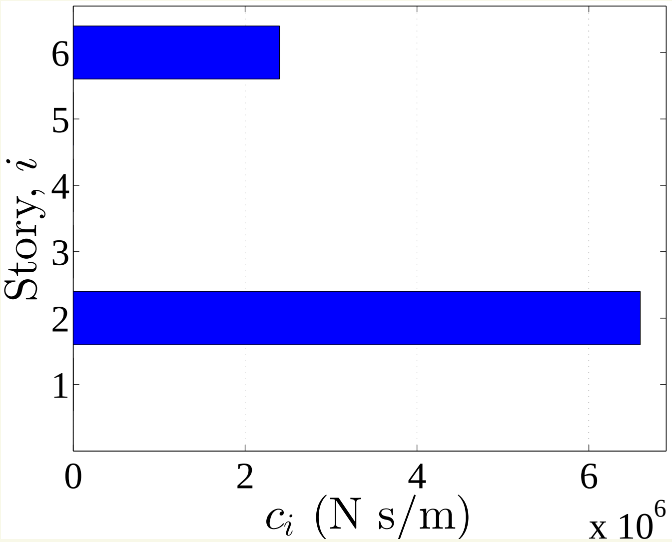
- no adjacent damped stories

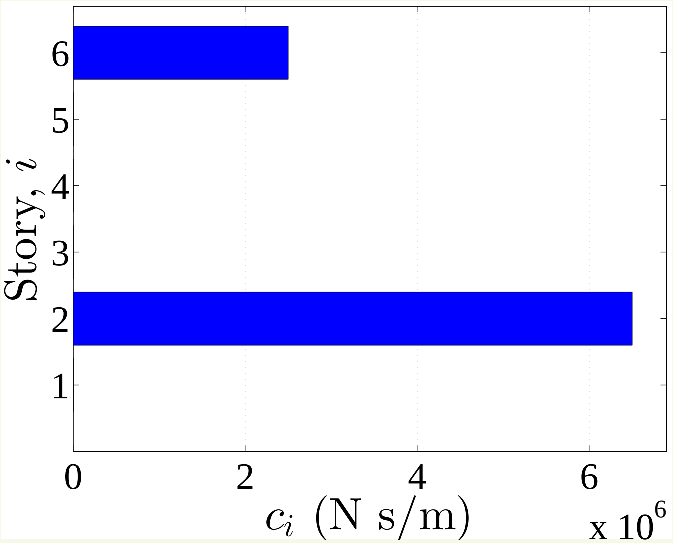
ex. 2) with more constraints



(a) $c_i \in \{0, 100, \ldots, 6000\}$ kNs/m



(b) $c_i \in \{0, 200, \ldots, 6000\}$ kNs/m



(c) $c_i \in \{0, 500, \ldots, 7500\}$ kNs/m

- upr. bd. for # of dampers: $\gamma = 2$
- no adjacent damped stories

conclusions

- optimal damper placement
  - supplemental viscous dampers
  - in a shear building model
- min. of transfer fcn. of interstory drifts
  - Minimize $\sum_{i=1}^{n} |\hat{\delta}_i|$ / Minimize $\max\{|\hat{\delta}_1|, \dots, |\hat{\delta}_n|\}$
- damper damping coeffs.: discrete design variables
  - chosen among available candidates
    - manufacturing and commercial convenience
    - combinatorial constraints on damper placement
- global optimization
  - mixed integer programming with second-order cone constraints