Robust Truss Topology Optimization under Uncertain Loads by Using Penalty Convex-Concave Procedure

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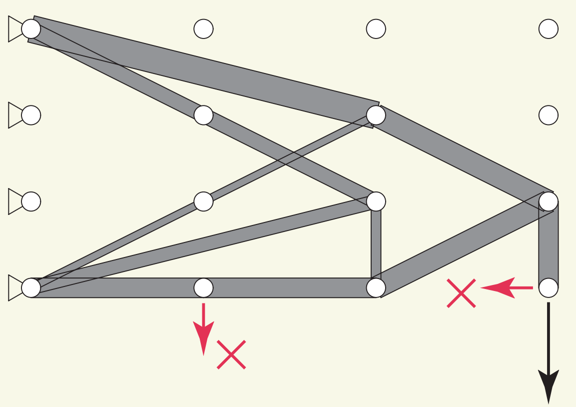
robustness against uncertainty in external load

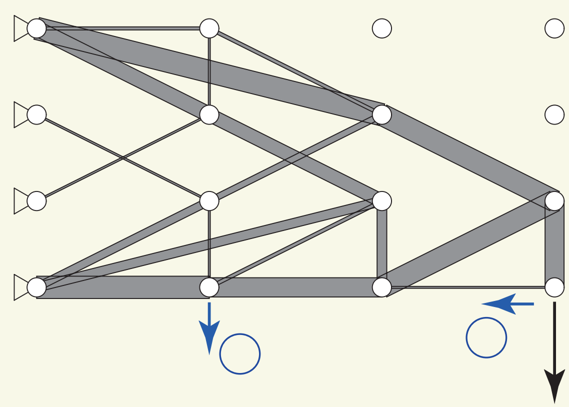
- robust truss topology optimization
  - a new modeling
    - semidefinite programming w/ complementarity constraints
  - an efficient heuristic
    - CCCP (concave-convex procedure)

robust topology optimization

- against uncertainty in external load
  - continuum [Cherkaev & Cherkaev '03, '08], [Guo, Bai, Zhang, Gao '09] [de Gournay, Allaire, & Jouve '08], [Guo, Du, & Gao '11] [Takezawa, Nii, Kitamura, & Kogiso '11], [Holmberg, Thore, & Klarbring '15]
  - truss [Ben-Tal & Nemirovski '97], [Yonekura & K. '10], [K. & Guo '10]
- methodology
  - specify set of uncertain loads
  - minimize worst-case compliance



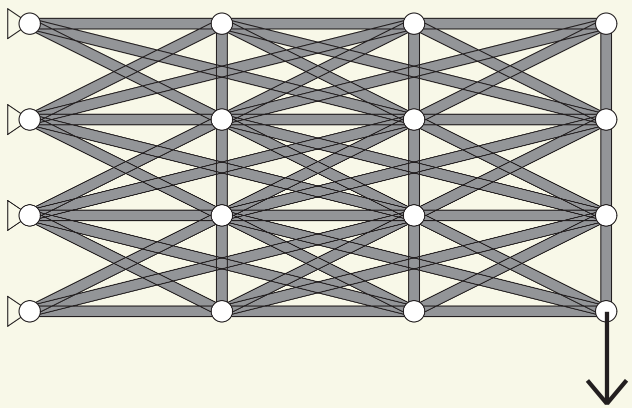
conventional opt.



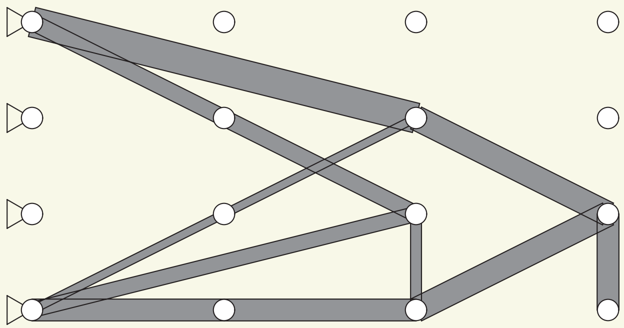
robust opt.

robust truss topology optimization: revisited

- min. worst-case compliance [Ben-Tal & Nemirovski '97]
  - uncertain loads at all nodes, in all directions



ground structure (GS)



conventional (nominal) opt.

robust truss topology optimization: revisited

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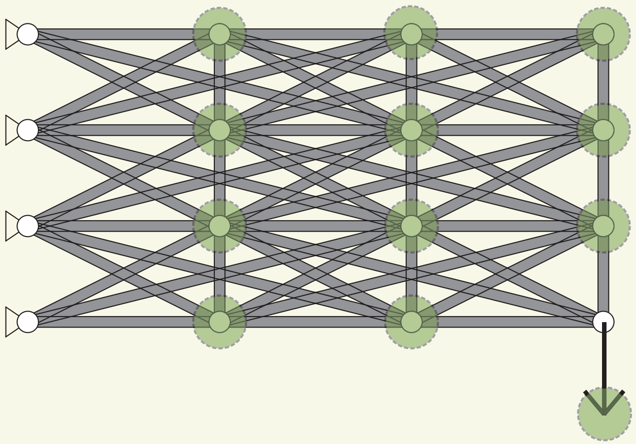

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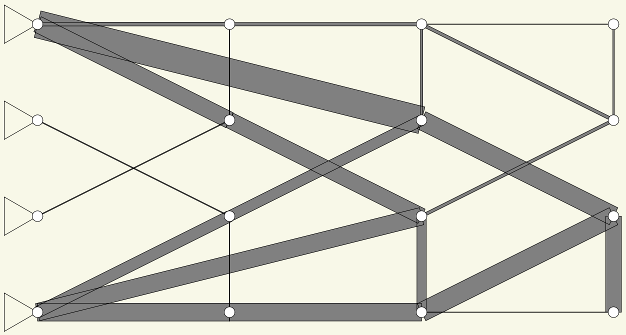

robust opt.

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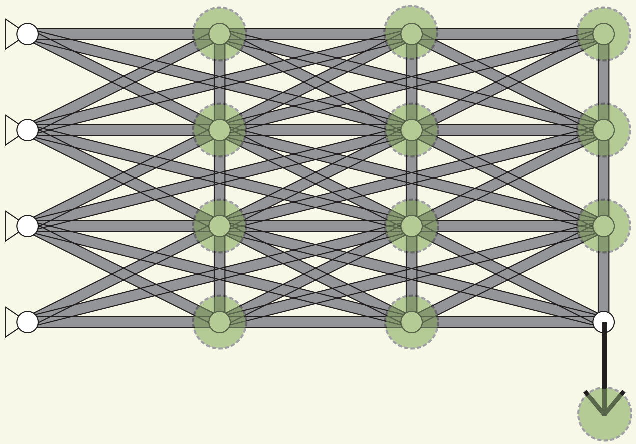

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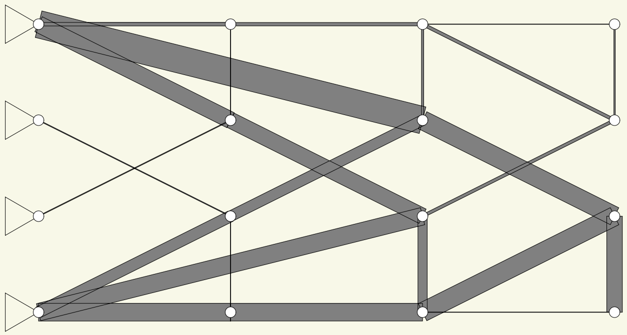
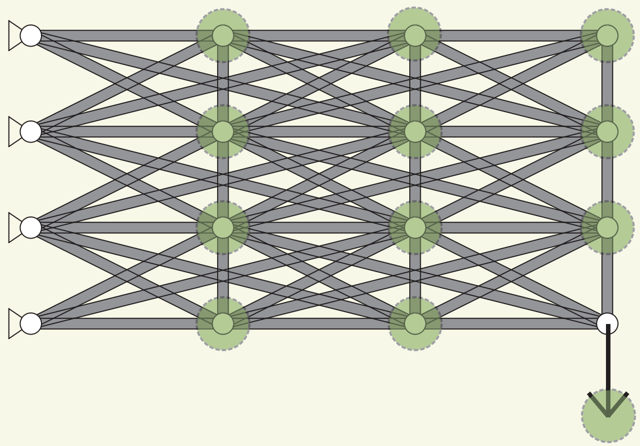


robust opt.

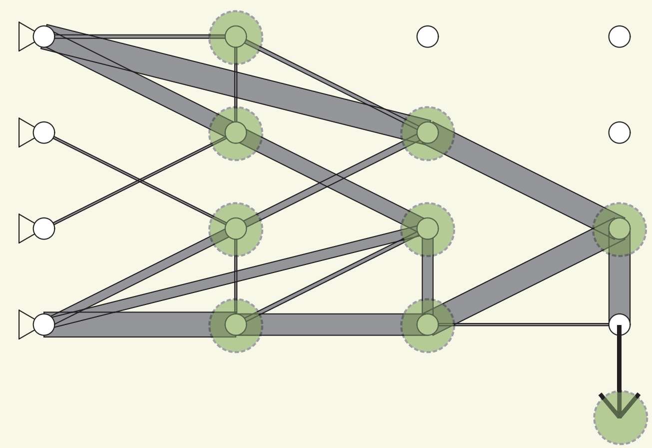
- design-independent uncertainty model
  - all nodes remain $\Rightarrow$ topology is not optimized

robust truss topology optimization: revisited

- min. worst-case compliance [Ben-Tal & Nemirovski '97]
  - uncertain loads at all nodes, in all directions



ground structure (GS)



robust optimal topology

- design-independent uncertainty model
  - all nodes remain $\Rightarrow$ topology is not optimized
- $\rightarrow$ topology-dependent uncertainty model

[Yonekura & K. '10], [K. & Guo '10]

robust truss topology optimization: revisited

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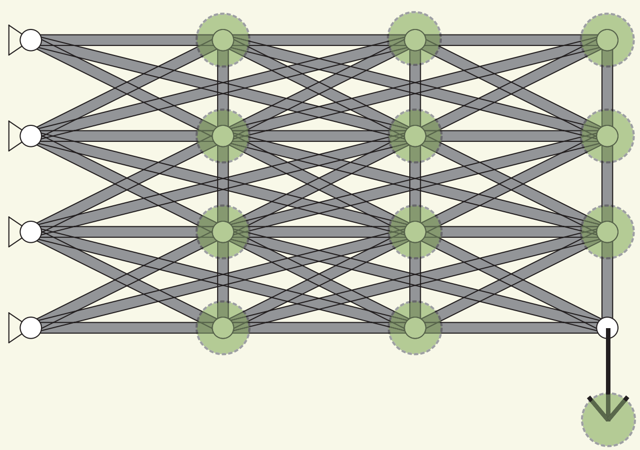
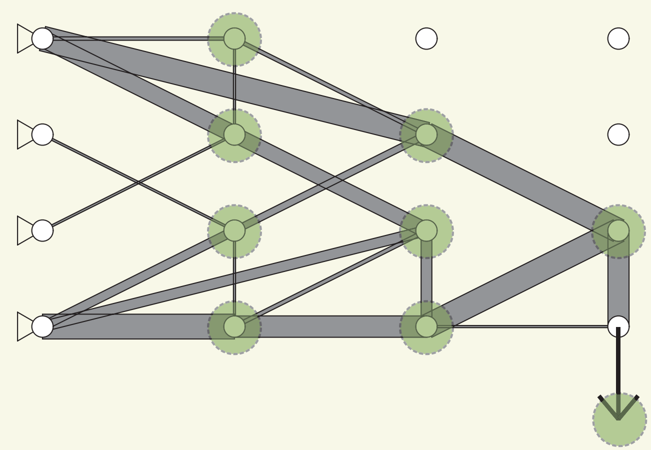
  - uncertain loads at all nodes, in all directions



ground structure (GS)



robust optimal topology???

- design-independent uncertainty model
  - all nodes remain ⇒ topology is not optimized
- → topology-dependent uncertainty model

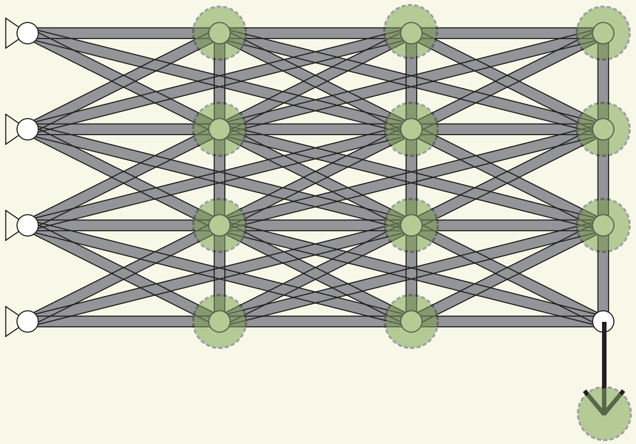
[Yonekura & K. '10], [K. & Guo '10]

uncertainty model

- ellipsoidal uncertainty [Ben-Tal & Nemirovski ’97]

$$\bar{\mathcal{F}} = \{Q\boldsymbol{e} \mid 1 \geq \|\boldsymbol{e}\|\} \qquad (Q\text{:constant matrix})$$

  - $\rightarrow$ semidefinite programming (SDP)
  - uncertain force at all nodes



uncertainty model

- ellipsoidal uncertainty [Ben-Tal & Nemirovski '97]

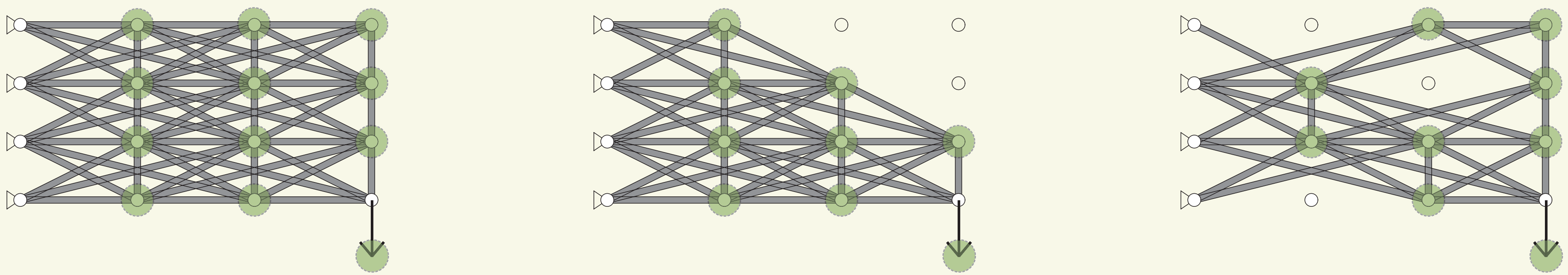
$$\bar{\mathcal{F}} = \{Q\boldsymbol{e} \mid 1 \geq \|\boldsymbol{e}\|\} \qquad (Q\text{:constant matrix})$$

  - → semidefinite programming (SDP)

- topology-dependent model [Yonekura & K. '10], [K. & Guo '10]

$$\mathcal{F}(\boldsymbol{s}) = \{\mathrm{diag}(\boldsymbol{s})Q\boldsymbol{e} \mid 1 \geq \|\boldsymbol{e}\|\}$$

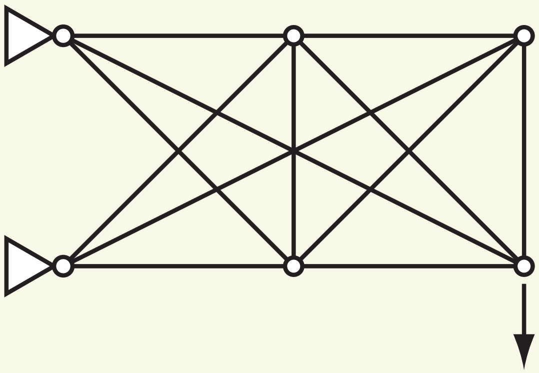
  - $s_j = \begin{cases} 1 & \text{if the } j\text{th DOF exists} \\ 0 & \text{if the } j\text{th DOF is removed} \end{cases}$



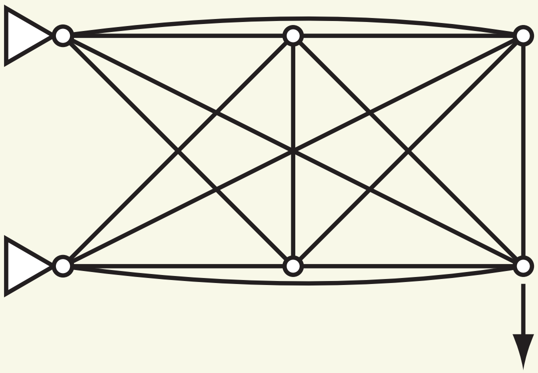
  - → mixed-integer semidefinite programming (MISDP)

more on topology optimization

- on overlapping members in ground structure



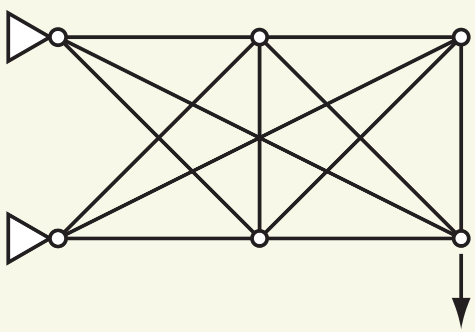
GS w/o overlapping members



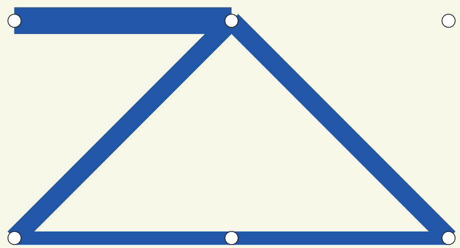
GS w/ overlapping members

chain & hinge cancelation

- nominal (=conventional) compliance minimization
  - use GS w/o overlapping members



GS

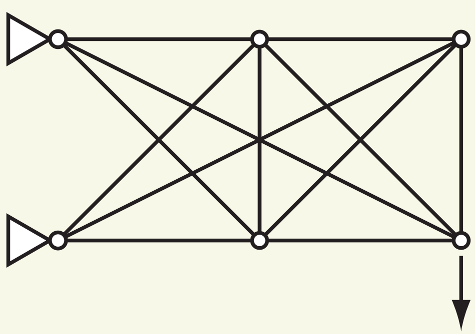


optimal solution

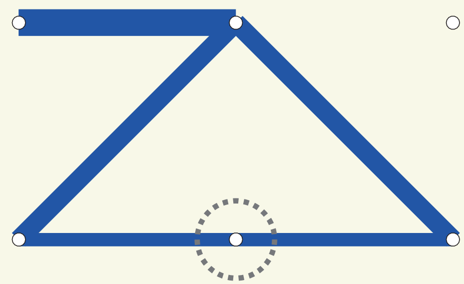
≡

chain & hinge cancelation

- nominal (=conventional) compliance minimization
  - use GS w/o overlapping members

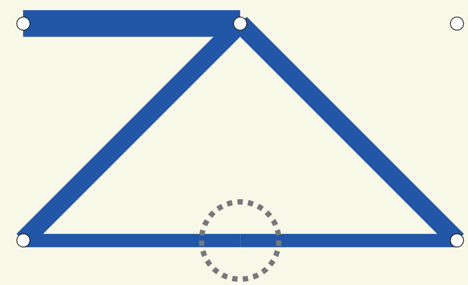


GS



optimal solution (having a chain)

≡

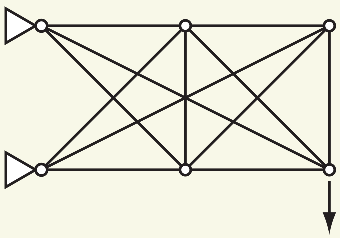


hinge cancelation

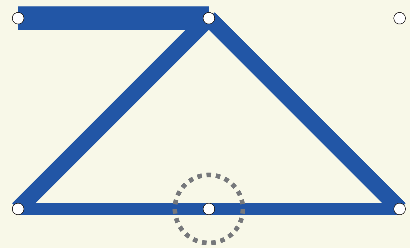
- chain:
  - a set of sequential parallel members
- hinge cancelation:
  - replace a chain by a single member
  - no change in objective value

nominal vs. robust optimization

- nominal opt.

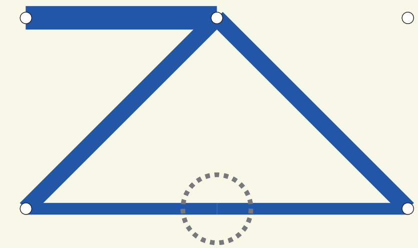


GS w/o overlapping



optimal solution

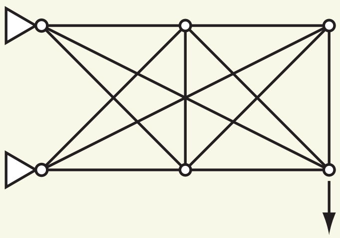
≡



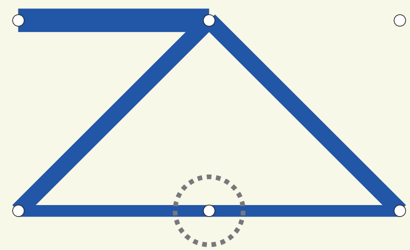
hinge cancelation

nominal vs. robust optimization

- nominal opt.

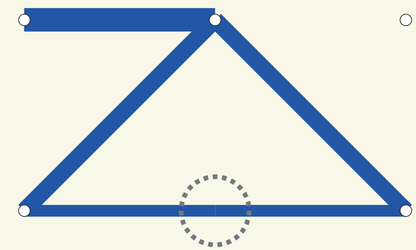


GS w/o overlapping



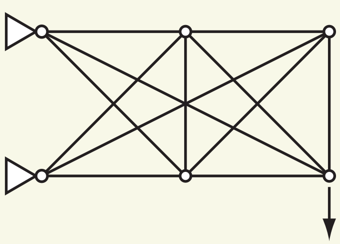
optimal solution

≡

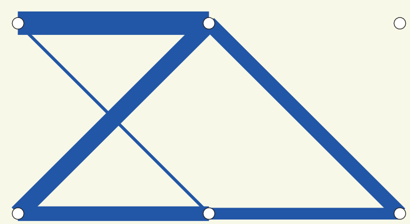


hinge cancelation

- robust opt.
  - GS w/o overlapping



→

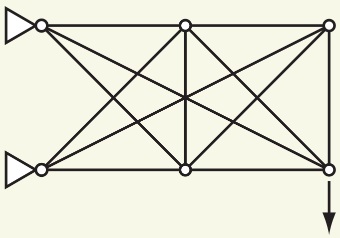


optimal

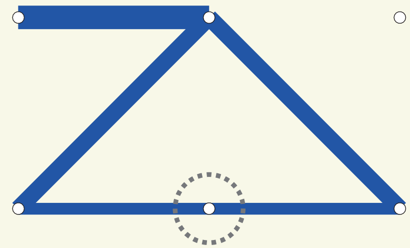
- The chain is stabilized w/ an additional bar.

nominal vs. robust optimization

- nominal opt.

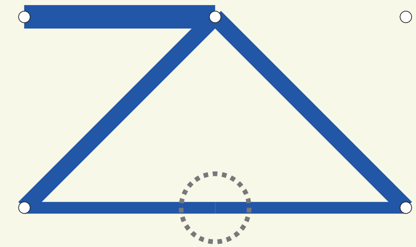


GS w/o overlapping



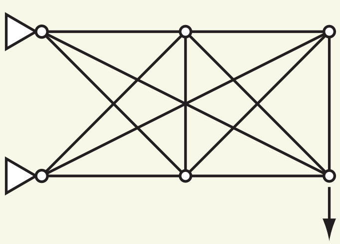
optimal solution

≡



hinge cancelation

- robust opt.
  - GS w/o overlapping



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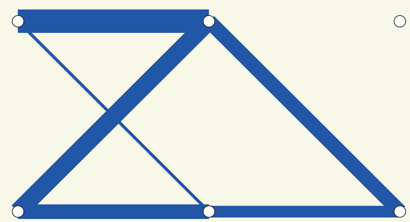
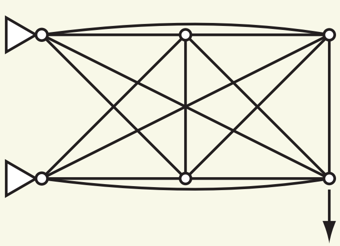


optimal???

- The chain is stabilized w/ an additional bar.

  - GS w/ overlapping



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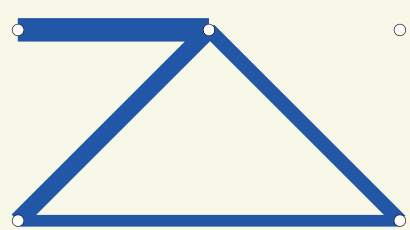
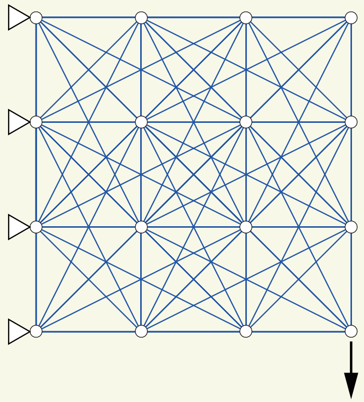


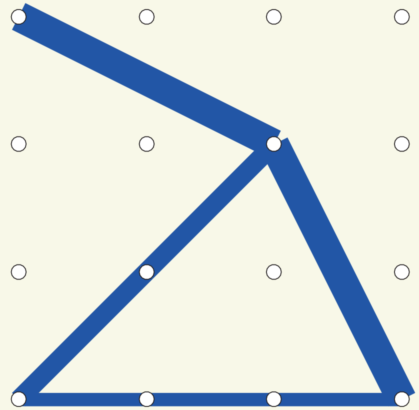
truly optimal

- The single long bar is chosen instead of the chain.

more ex. on overlapping members in GS

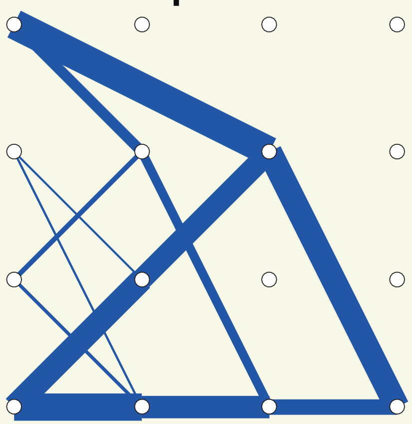
- problem setting



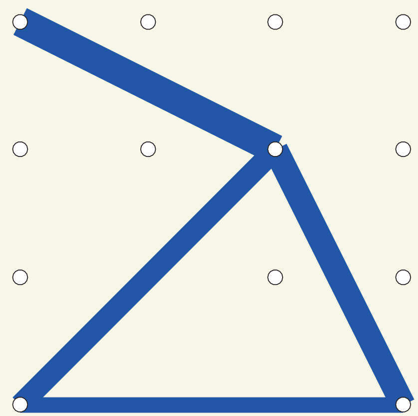


nominal opt.

- robust opt.



from GS w/o overlapping
$obj = 3259.1$ J

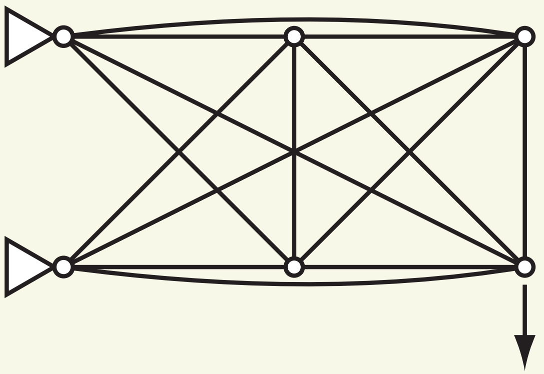


from GS w/ overlapping
$obj = 2442.7$ J

- uncertain load: at all existing nodes
- Overlapping in a solution is prohibited.
- GS w/ overlapping members yields a better solution.

new modeling

- robust truss topology opt.
  - GS should include overlapping members, but...
  - the solution should not include overlapping members.



new modeling (1/3)

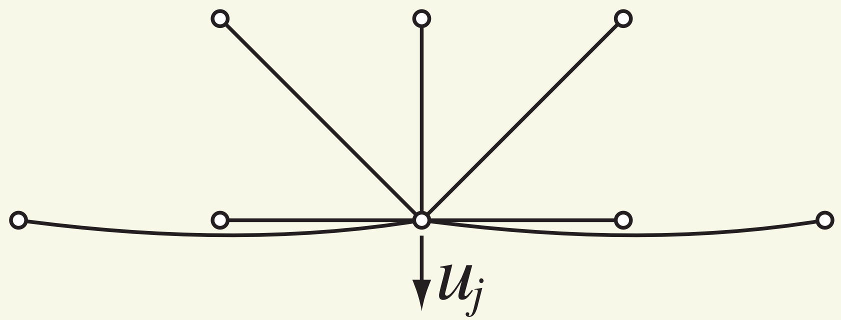
- multiplier for uncertain load $s_j \in [0, 1]$:

$$s_j = \begin{cases} 1 & \text{if the } j\text{th DOF exists} \\ 0 & \text{if the } j\text{th DOF is removed} \end{cases}$$

- sum of c-s areas of members connected to the jth DOF:

$$n_j = \sum_{e \in N(j)} x_e$$

  - x_e : member cross-sectional area
  - n_j :



- complementarity:

$$(1 - s_j) n_j = 0$$

new modeling (2/3)

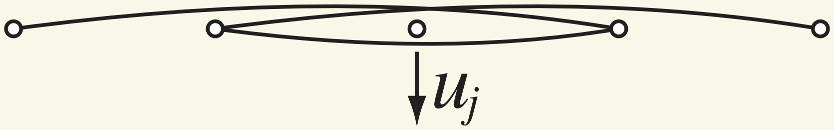
- multiplier for uncertain load $s_j \in [0, 1]$:

$$s_j = \begin{cases} 1 & \text{if the } j\text{th DOF exists} \\ 0 & \text{if the } j\text{th DOF is removed} \end{cases}$$

- sum of c-s areas of members lying across the jth DOF:

$$a_j = \sum_{e \in A(j)} x_e$$

  - x_e : member cross-sectional area
  - a_j :



- complementarity:

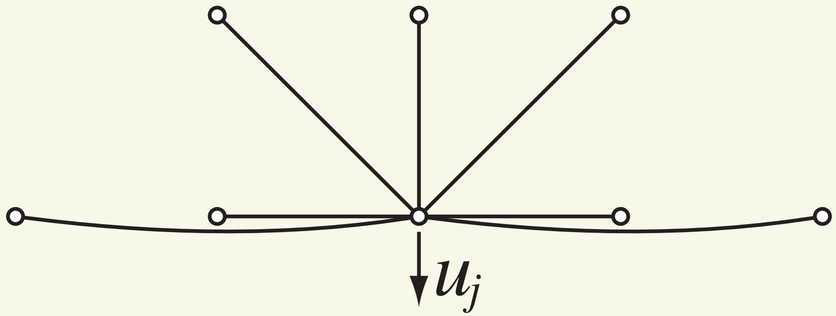
$$s_j a_j = 0$$

new modeling (3/3)

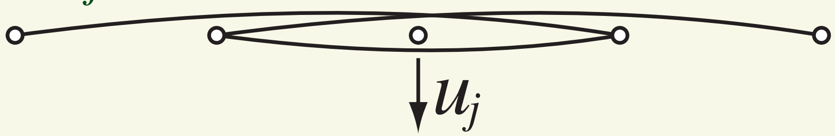
- complementarity:

$$(1 - s_j)n_j = 0, \quad s_j a_j = 0 \qquad (\spadesuit)$$

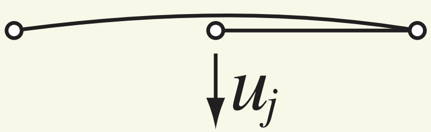
  - n_j :

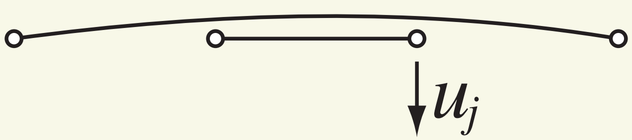


  - a_j :



- overlapping cases:





  - observation:

$$n_j > 0 \Rightarrow s_j = 1, \qquad a_j > 0$$

$\therefore$ ($\spadesuit$) is not satisfied.

upshot

- complementarity constraints:
  - representing topology-dependent uncertain loads
  - avoiding presence of overlapping members
- existing formulation (GS w/o overlapping)
  - convex opt. (semidefinite programming) [Ben-Tal & Nemirovski ’97]

upshot

- complementarity constraints:
  - representing topology-dependent uncertain loads
  - avoiding presence of overlapping members
- existing formulation (GS w/o overlapping)
  - convex opt. (semidefinite programming) [Ben-Tal & Nemirovski ’97]
- new formulation
  - convex opt. w/ complementarity constraints
    - ...difficult to solve globally...

a simple heuristic

- CCCP (concave-convex procedure)
  - also known as (and related to)...
    - convex-concave procedure
    - DCA (difference-of-convex algorithm) [Pham Dinh & Le Thi '97]
    - EM (expectation-maximization) algorithm [Dempster, Laird, & Rubin '77]
    - MM (majorization-minimization) algorithm
      - frequently used in machine learning & image processing [Hunter & Lange '00], [Figueiredo, Bioucas-Dias, & Nowak '07] [Sriperumbudur, Torres, & Lanckriet '11], [Sun, Babu, & Palomar '17]
  - a heuristic for DC (difference-of-convex) programming

concave-convex procedure for robust truss optimization

- convex optimization w/ complementarity constraints:

$$
\begin{array}{ll}
\text{Min.} & f(\boldsymbol{x}) \\
\text{s. t.} & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \Omega, \\
 & \boldsymbol{y}, \boldsymbol{z} \geq \boldsymbol{0}, \ \ \boldsymbol{y}^{\top}\boldsymbol{z} = 0.
\end{array}
$$

  - f : convex fctn. Ω : convex set

concave-convex procedure for robust truss optimization

- convex optimization w/ complementarity constraints:

$$\begin{aligned} \text{Min.} \quad & f(\boldsymbol{x}) \\ \text{s. t.} \quad & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \Omega, \\ & \boldsymbol{y}, \boldsymbol{z} \geq \boldsymbol{0}, \ \boldsymbol{y}^\top \boldsymbol{z} = 0. \end{aligned}$$

  - f : convex fctn. Ω : convex set

- penalized formulation: [Jara-Moroni, Pang, & Wächter '16]

$$\begin{aligned} \text{Min.} \quad & f(\boldsymbol{x}) + \rho \underbrace{(\|\boldsymbol{y} + \boldsymbol{z}\|^2 - \|\boldsymbol{y} - \boldsymbol{z}\|^2)}_{4\boldsymbol{y}^\top \boldsymbol{z}} \\ \text{s. t.} \quad & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \Omega, \\ & \boldsymbol{y}, \boldsymbol{z} \geq \boldsymbol{0}. \end{aligned}$$

  - ρ : penalty parameter (sufficiently large)

concave-convex procedure for robust truss optimization

- convex optimization w/ complementarity constraints:

$$\begin{aligned} \text{Min.} \quad & f(\boldsymbol{x}) \\ \text{s. t.} \quad & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \Omega, \\ & \boldsymbol{y}, \boldsymbol{z} \geq \boldsymbol{0}, \ \ \boldsymbol{y}^{\top}\boldsymbol{z} = 0. \end{aligned}$$

  - f : convex fctn. Ω : convex set

- penalized form—DC program [Jara-Moroni, Pang, & Wächter '16]

$$\begin{aligned} \text{Min.} \quad & \underbrace{f(\boldsymbol{x}) + \rho\|\boldsymbol{y} + \boldsymbol{z}\|^2}_{\text{convex}} - \underbrace{\rho\|\boldsymbol{y} - \boldsymbol{z}\|^2}_{\text{convex}} \\ \text{s. t.} \quad & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \Omega, \\ & \boldsymbol{y}, \boldsymbol{z} \geq \boldsymbol{0}. \end{aligned}$$

  - ρ : penalty parameter (sufficiently large)
  - DC = "difference of convex"

concave-convex procedure for robust truss optimization

- convex optimization w/ complementarity constraints:

$$\begin{aligned} \text{Min.} \quad & f(\boldsymbol{x}) \\ \text{s. t.} \quad & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \Omega, \\ & \boldsymbol{y}, \boldsymbol{z} \geq \boldsymbol{0}, \ \boldsymbol{y}^\top \boldsymbol{z} = 0. \end{aligned}$$

  - f : convex fctn. Ω : convex set

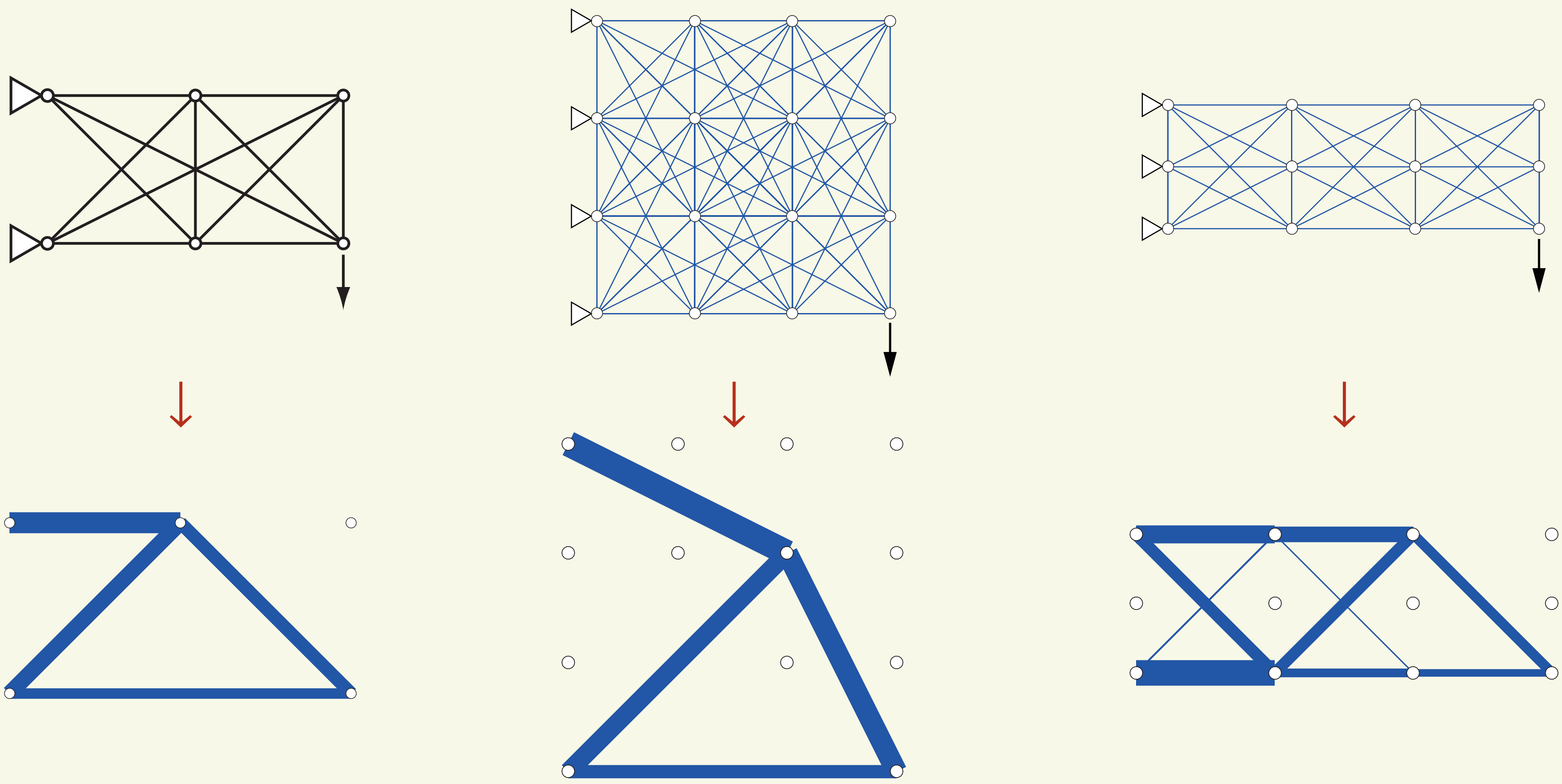
- proposed method:

$$\begin{aligned} \text{Min.} \quad & \underbrace{f(\boldsymbol{x}) + \rho_k \|\boldsymbol{y} + \boldsymbol{z}\|^2}_{\text{convex}} - \underbrace{\rho_k \|\boldsymbol{y} - \boldsymbol{z}\|^2}_{\text{linearize at } (\boldsymbol{y}_k, \boldsymbol{z}_k)} \\ \text{s. t.} \quad & (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \Omega, \\ & \boldsymbol{y}, \boldsymbol{z} \geq \boldsymbol{0}. \end{aligned}$$

  - at each iteration: solve a convex subproblem
  - ρ_k : gradually increased

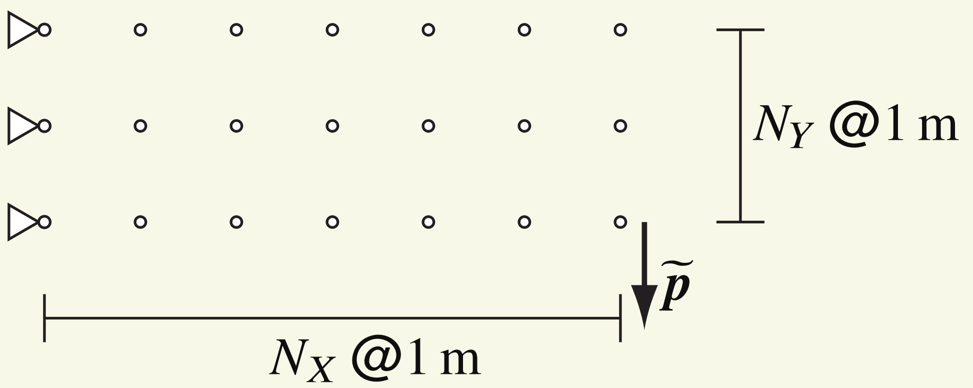
num. expt. (1): small-size instances

- comparison w/ branch-and-bound method (YALMIP)
  - GS w/ overlapping members
  - The proposed method converges to global opt.



num. expt. (2)

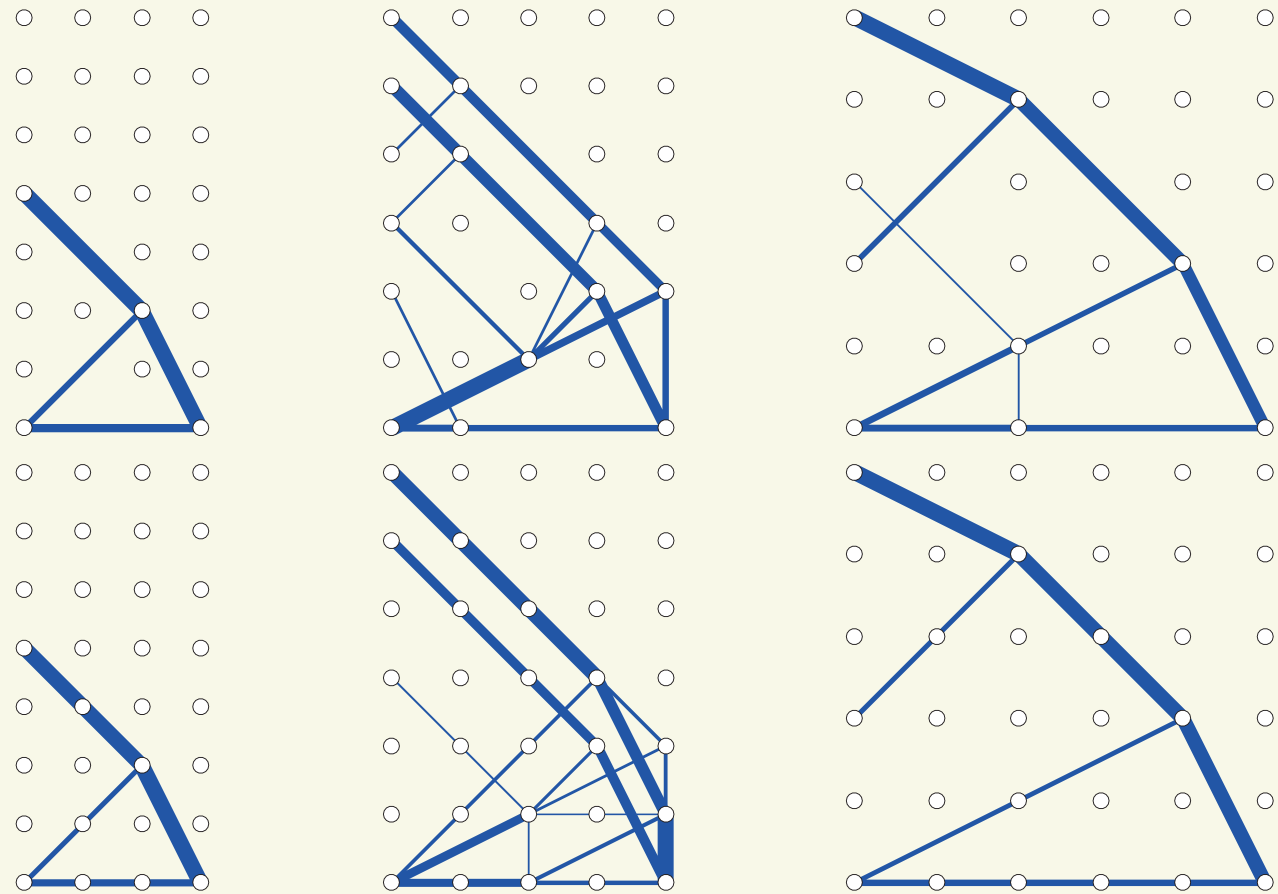
- problem setting



- uncertainty in external load
  - $\tilde{\boldsymbol{p}}$: nominal load
- avoiding thin members: $x_i \in \{0\} \cup [\underline{x}, \overline{x}]$
- avoiding too long members:
  - "Members $> 3\,\mathrm{m}$" were in advance removed from GS.

num. expt. (2)

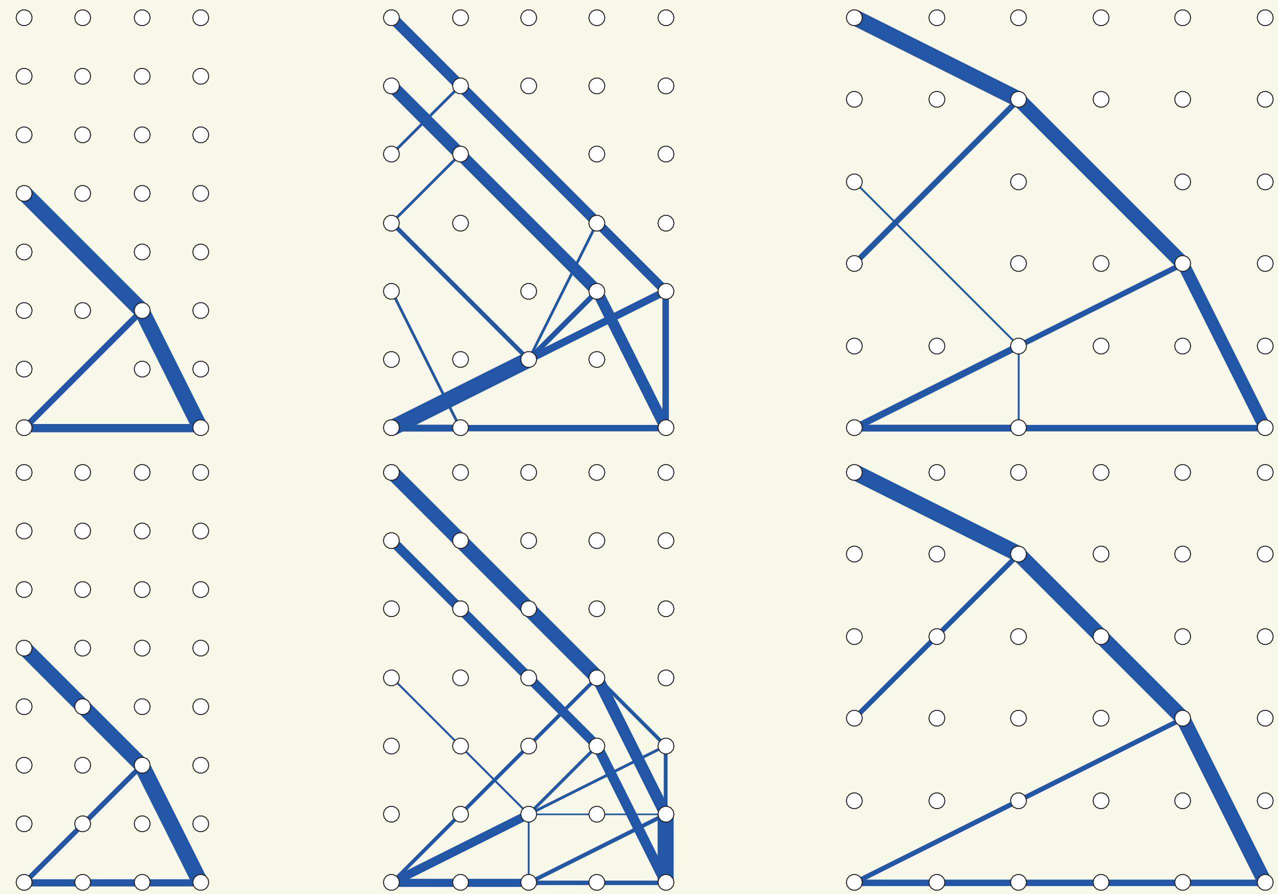
- robust opt.
- nominal opt.



- Nominal opt.:
  - possibly has a very long chain.

num. expt. (2)

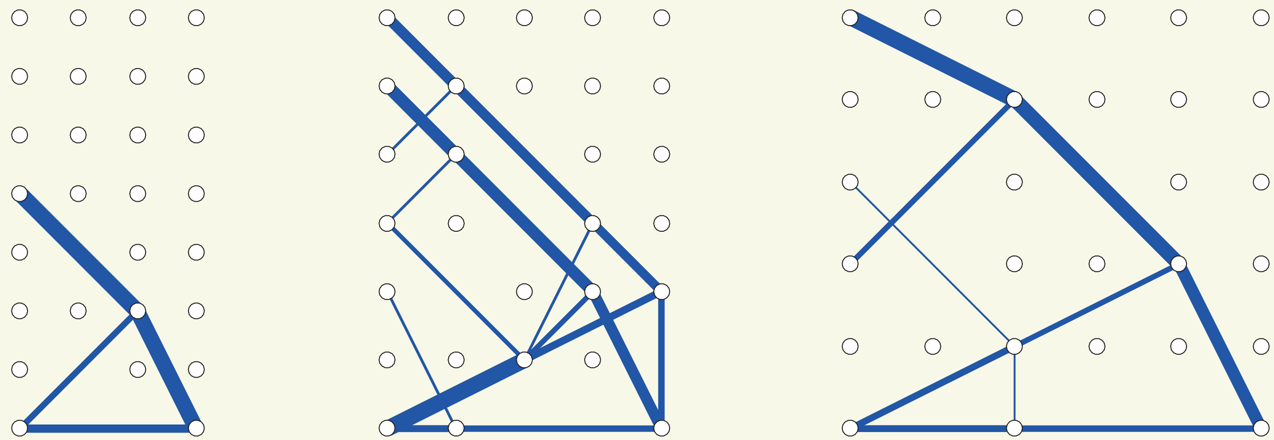
- robust opt.
- nominal opt.



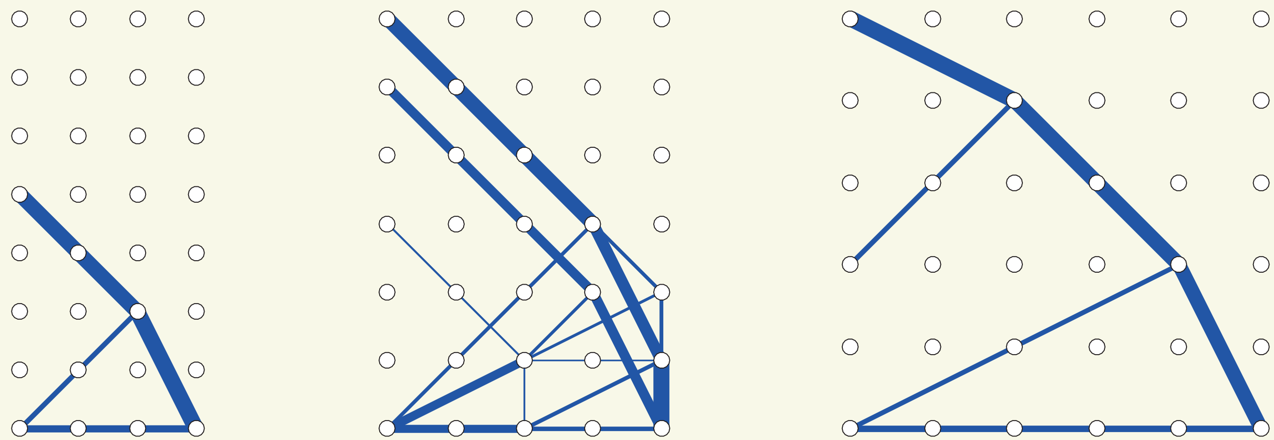
- In robust opt.:
  - Chains are replaced by long single members.
  - (max. mbr. length in sol.) $\leq$ (max. mbr. length in GS) $= 3\,\mathrm{m}$.
    - Otherwise, infeasible.

num. expt. (2)

- robust opt.



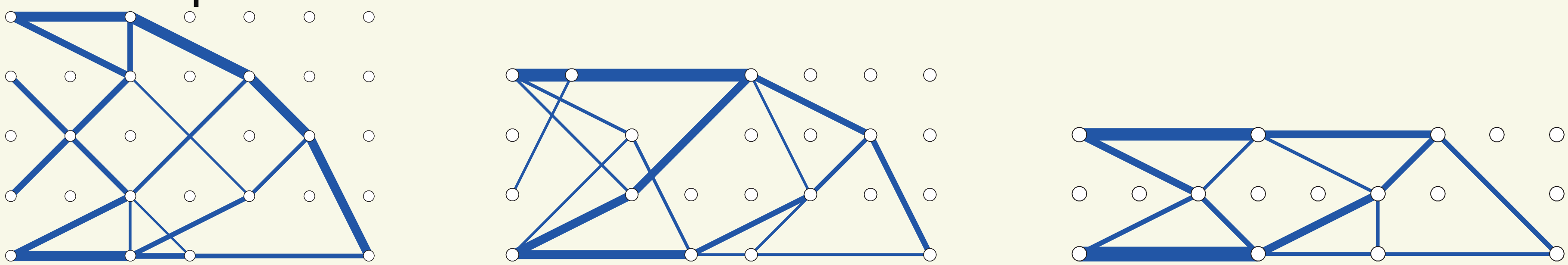
- nominal opt.



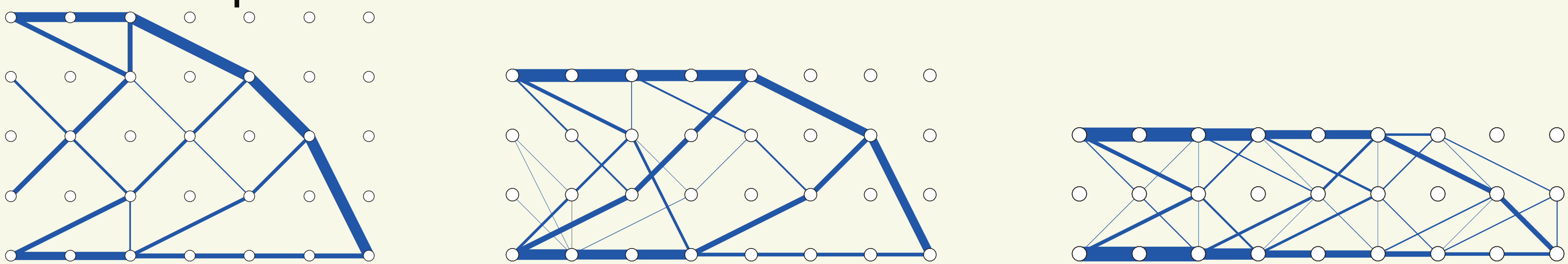
(N_X, N_Y)	#mbr.	#iter.	Time (s)
(3, 7)	250	9	46.4
(4.6)	292	39	242.9
(5, 5)	306	35	249.6

num. expt. (2)

- robust opt.



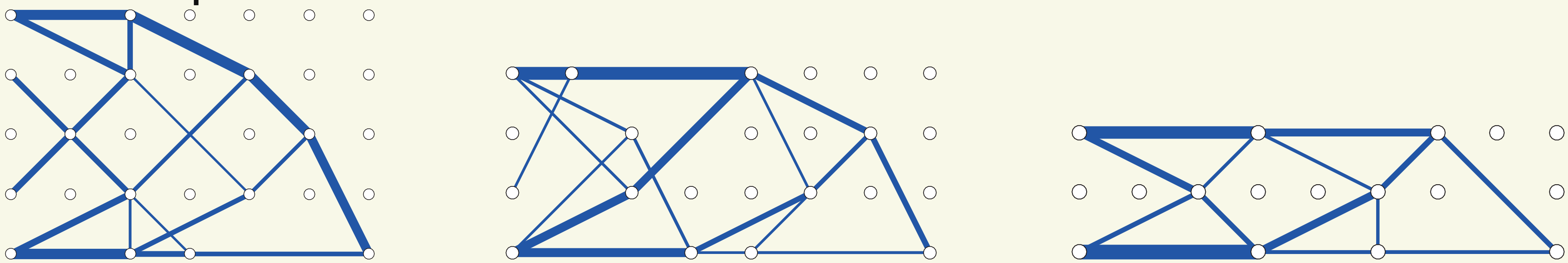
- nominal opt.



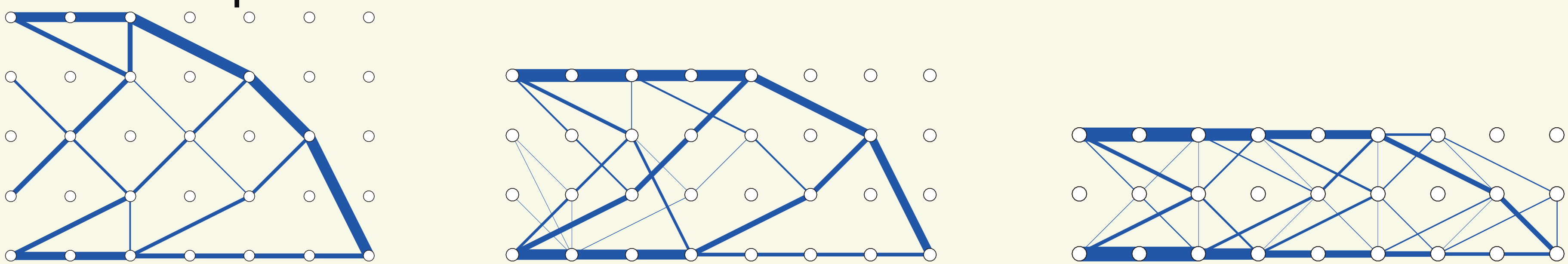
- Nominal opt.:
  - may have long chains.
  - may have very thin members.

num. expt. (2)

- robust opt.



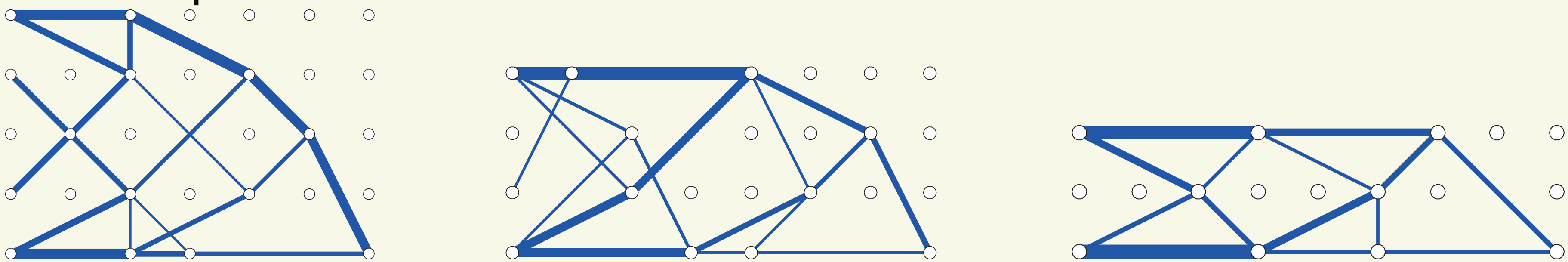
- nominal opt.



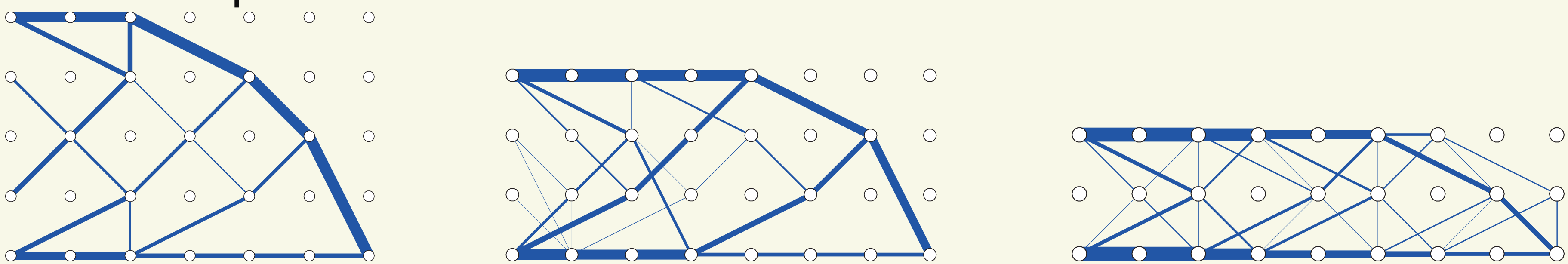
- In robust opt.:
  - Chains are replaced by long single members.
  - (max. mbr. length in sol.) $\leq$ (max. mbr. length in GS) $= 3\,\mathrm{m}$.
  - No thin member ($\because$ lwr. bnd. cstr. for mbr. area).

num. expt. (2)

- robust opt.



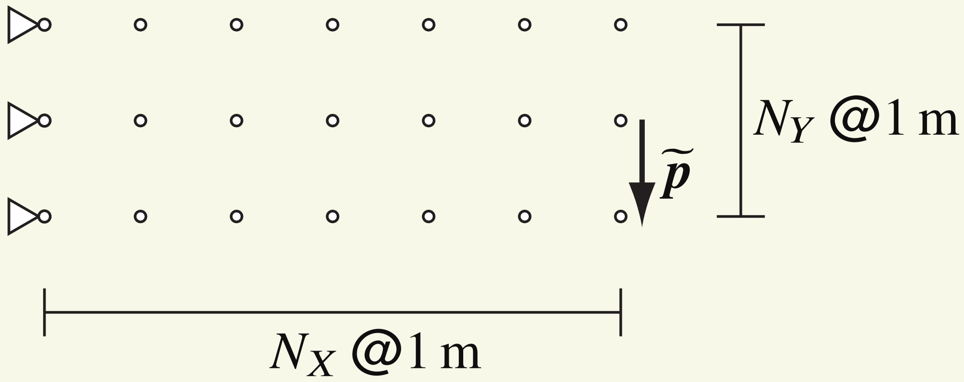
- nominal opt.



(N_X, N_Y)	#mbr.	#iter.	Time (s)
(6, 4)	292	21	142.1
(7.3)	250	40	193.9
(8, 2)	180	32	103.9

num. expt. (3)

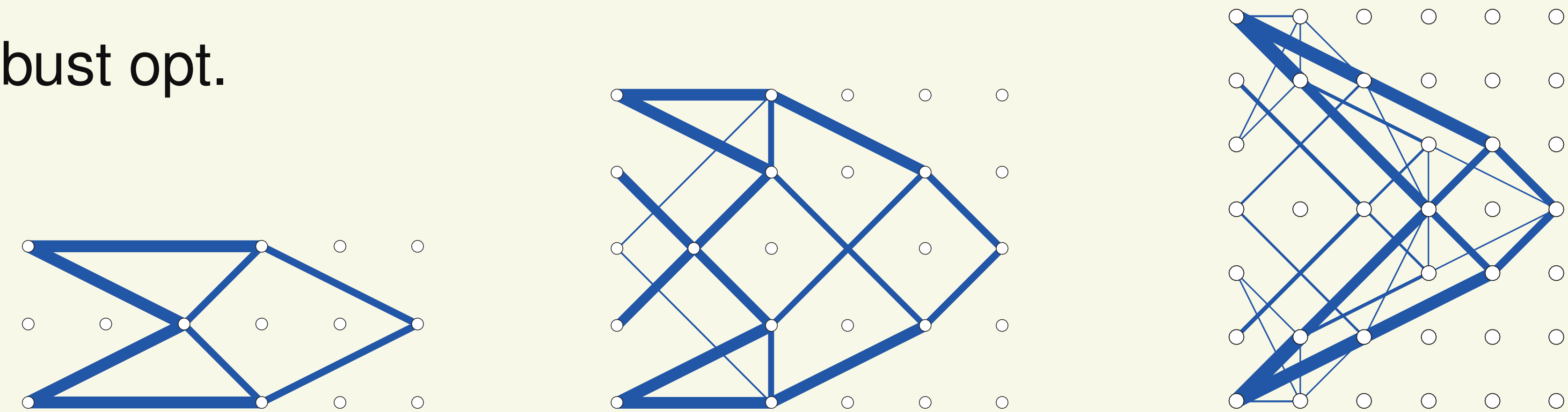
- problem setting



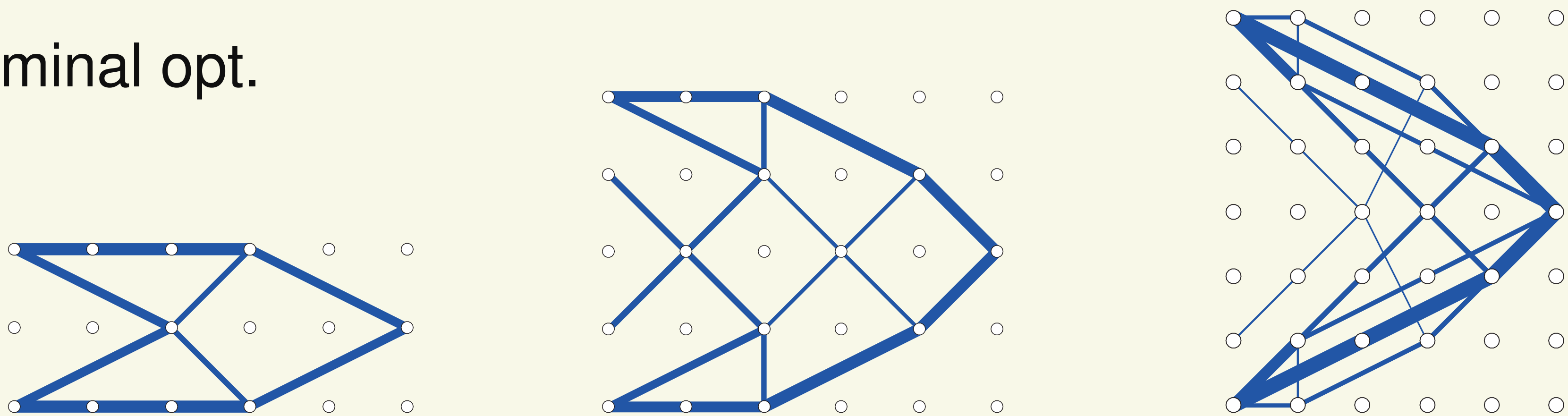
- uncertainty in external load
  - $\tilde{\boldsymbol{p}}$: nominal load
- avoiding thin members: $x_i \in \{0\} \cup [\underline{x}, \overline{x}]$
- avoiding too long members:
  - "Members $> 3\,\mathrm{m}$" were in advance removed from GS.

num. expt. (3)

- robust opt.



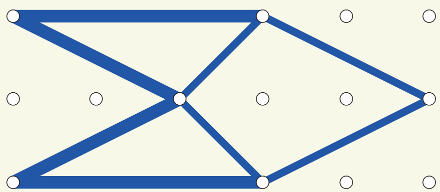
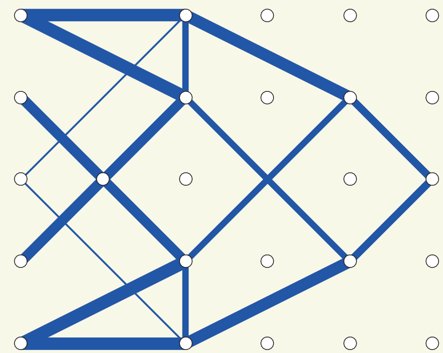
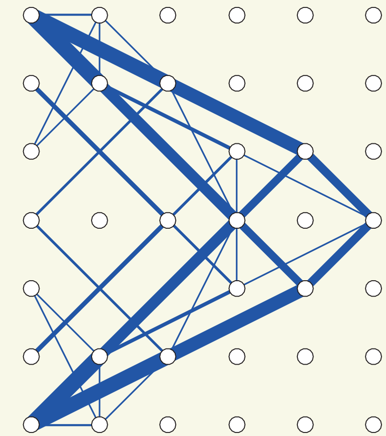
- nominal opt.



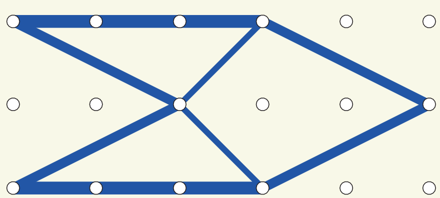
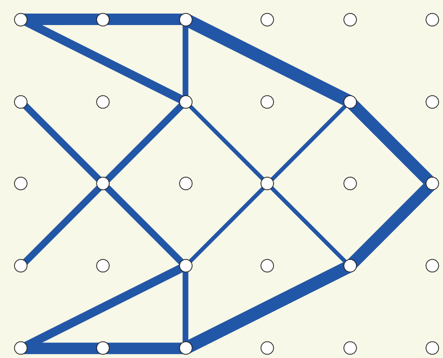
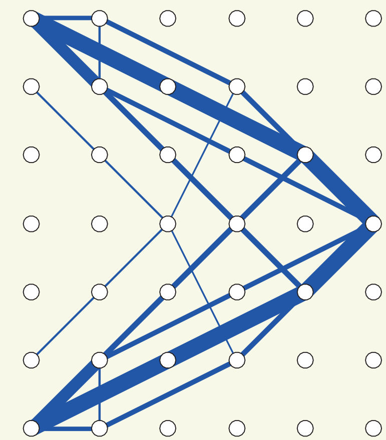
- In robust opt.:
  - Chains are replaced by single members.

num. expt. (3)

- robust opt.

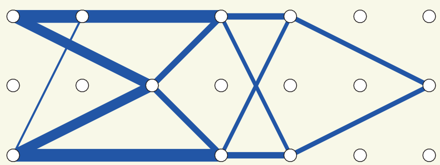
- nominal opt.

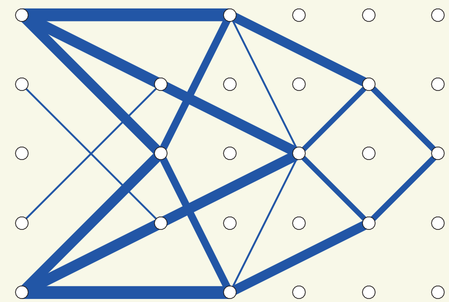




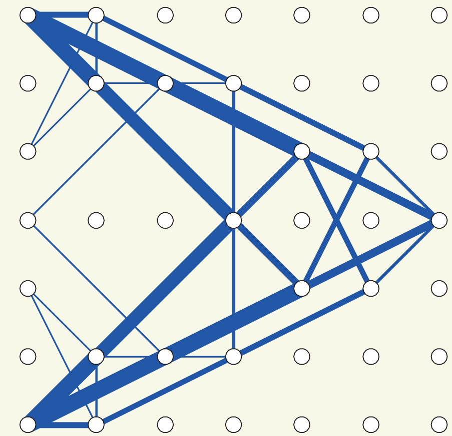
(N_X, N_Y)	#memb.	#iter.	Time (s)
$(5, 2)$	108	18	25.0
$(5, 4)$	240	16	62.1
$(5, 6)$	372	26	212.8

num. expt. (3)

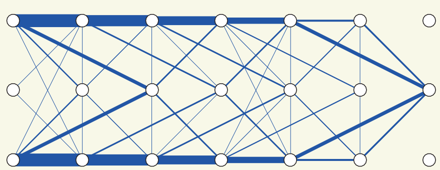
- robust opt.

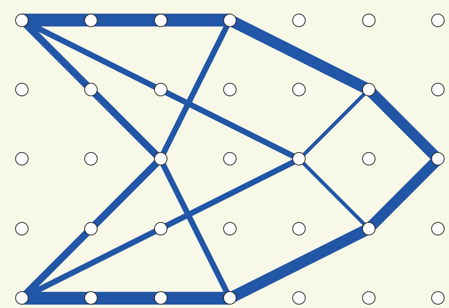


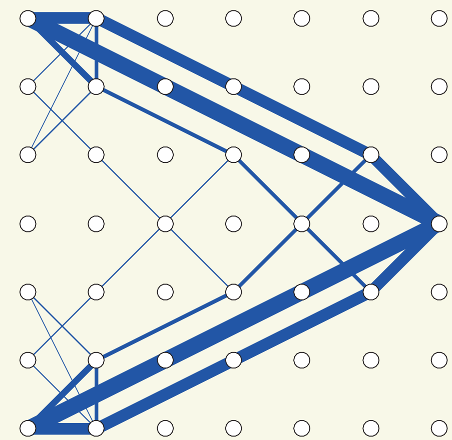




- nominal opt.



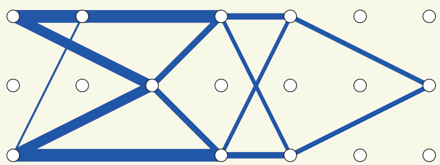


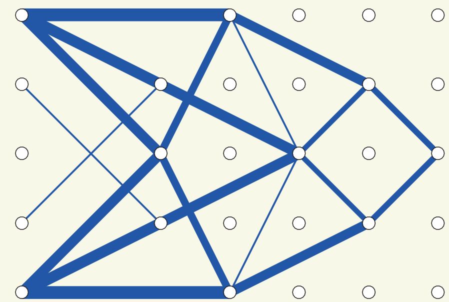


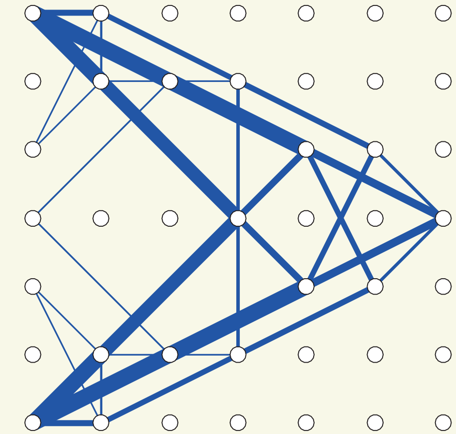
- In robust opt.:
  - Chains are replaced by single members.
  - Too thin members are disappeared.
  - Less nodes means less uncertain external forces.

num. expt. (3)

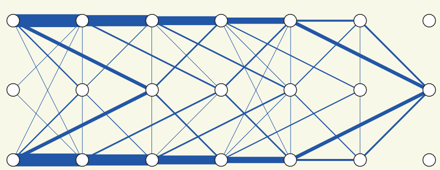
- robust opt.

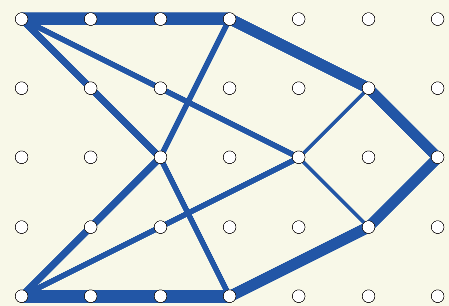


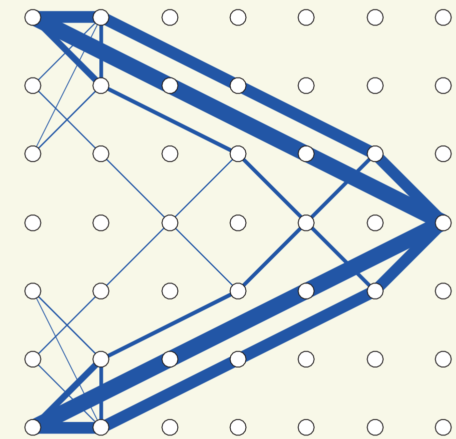




- nominal opt.



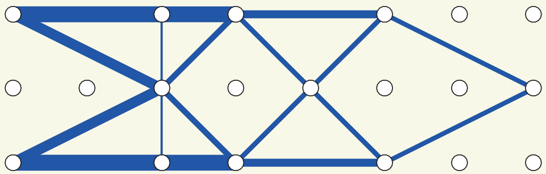


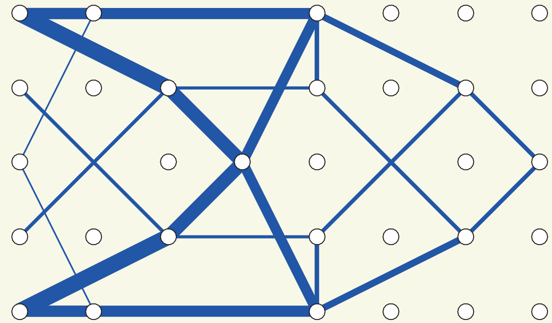


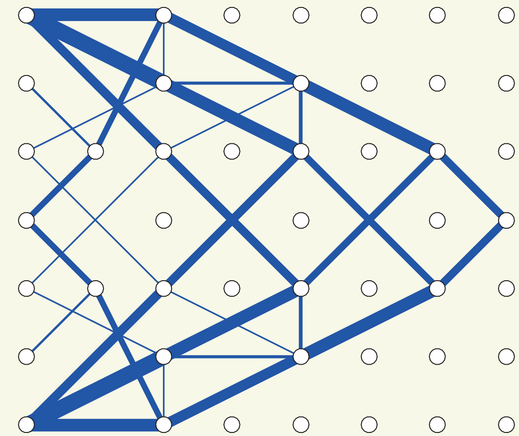
(N_X, N_Y)	#memb.	#iter.	Time (s)
(6, 2)	132	43	70.4
(6, 4)	292	19	113.9
(6, 6)	452	19	208.0

num. expt. (3)

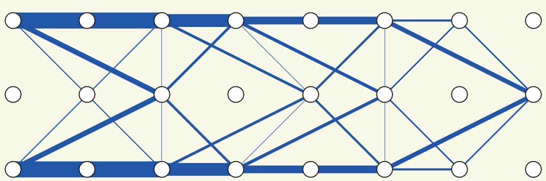
- robust opt.

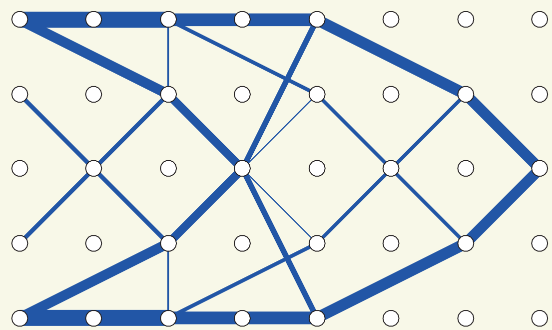


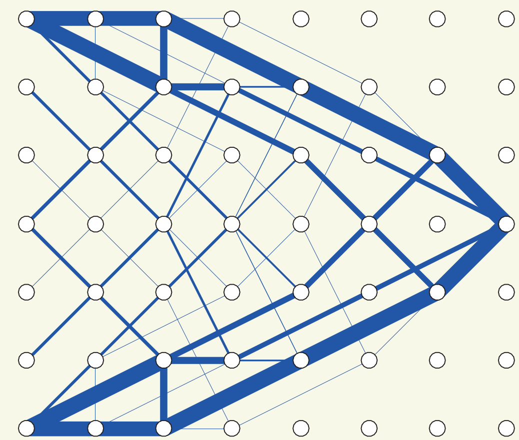




- nominal opt.



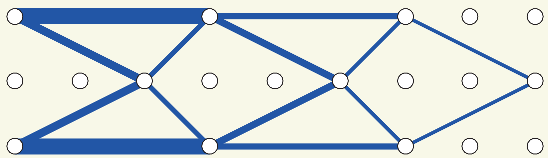
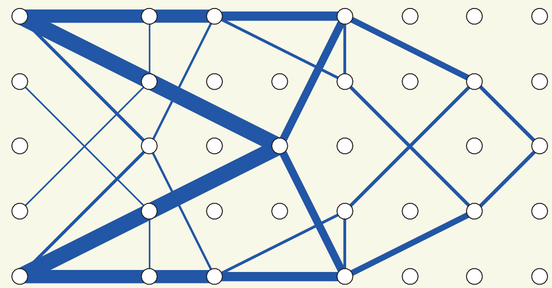
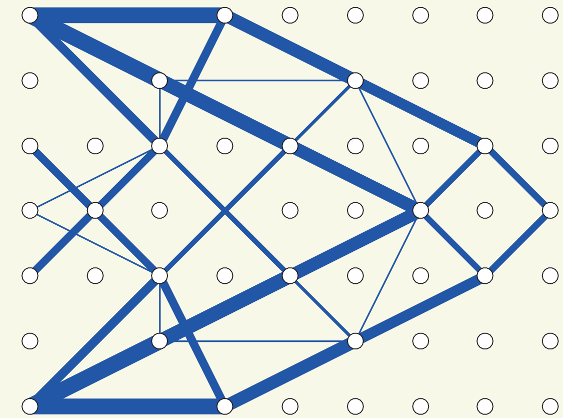




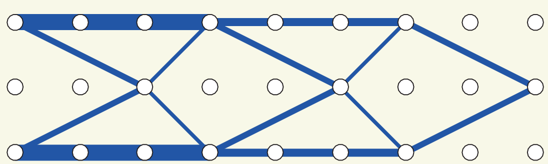
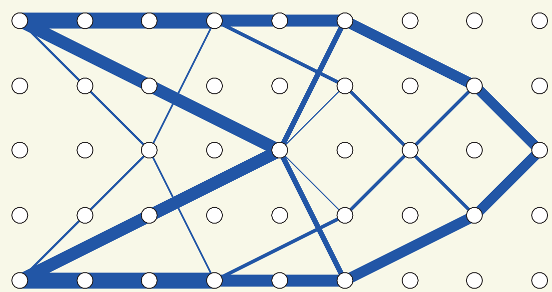
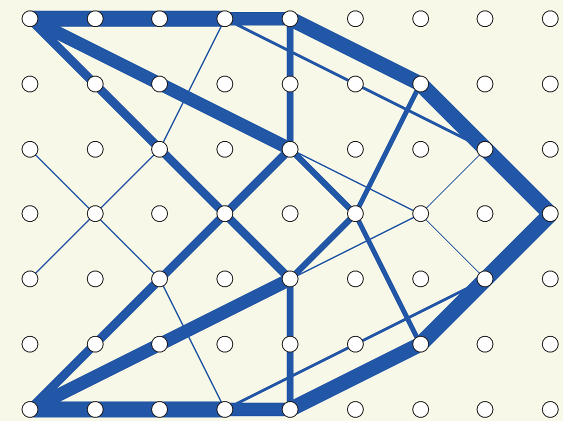
(N_X, N_Y)	#memb.	#iter.	Time (s)
$(7, 2)$	156	18	87.7
$(7, 4)$	344	40	320.6
$(7, 6)$	532	15	226.7

num. expt. (3)

- robust opt.

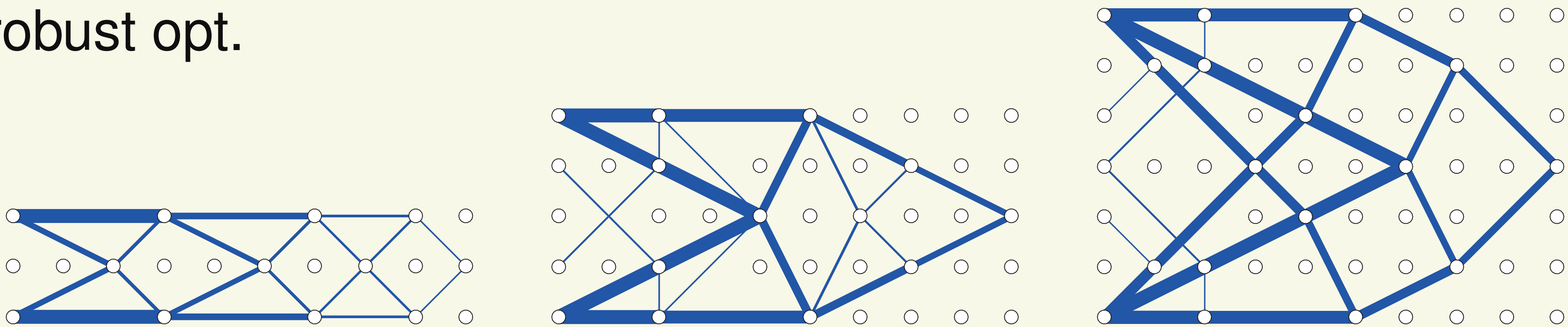
- nominal opt.

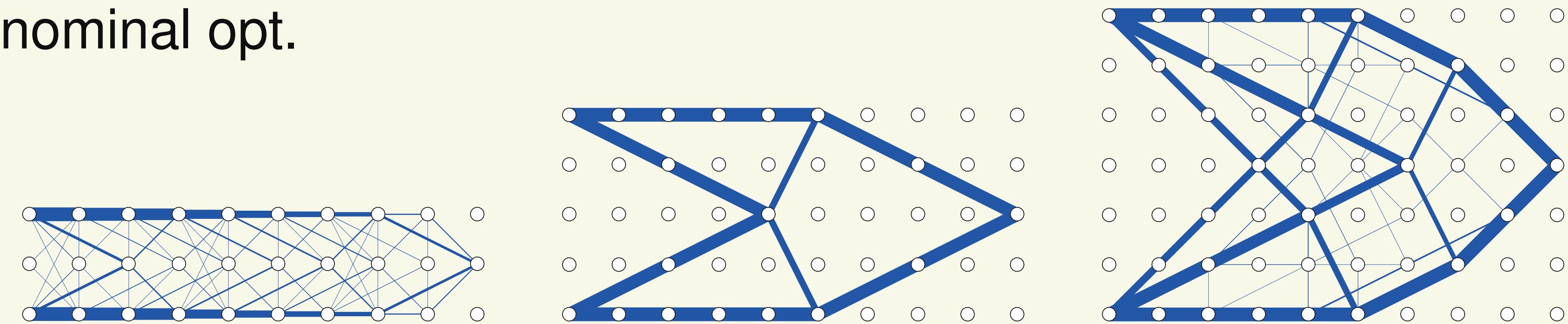
(N_X, N_Y)	#memb.	#iter.	Time (s)
(8, 2)	180	23	64.5
(8, 4)	396	37	440.0
(8, 6)	612	32	632.0

num. expt. (3)

- robust opt.



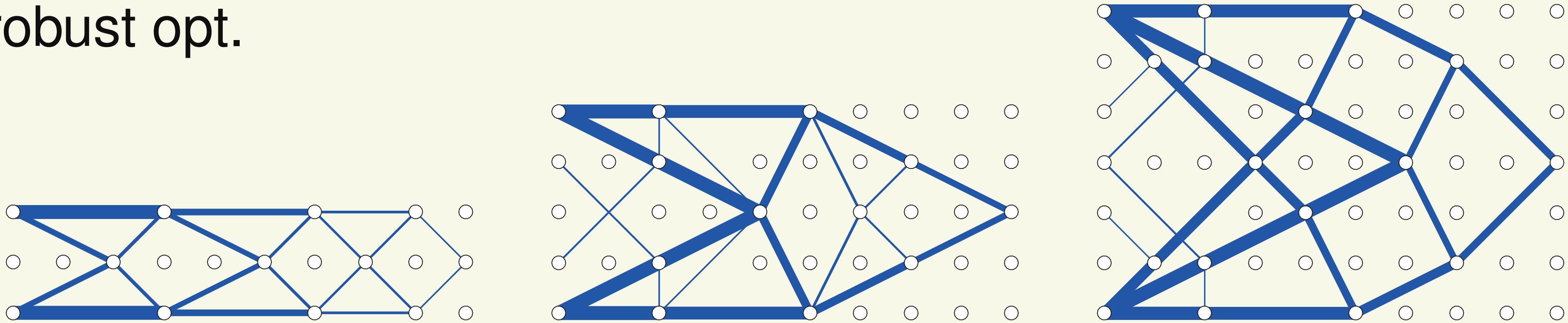
- nominal opt.



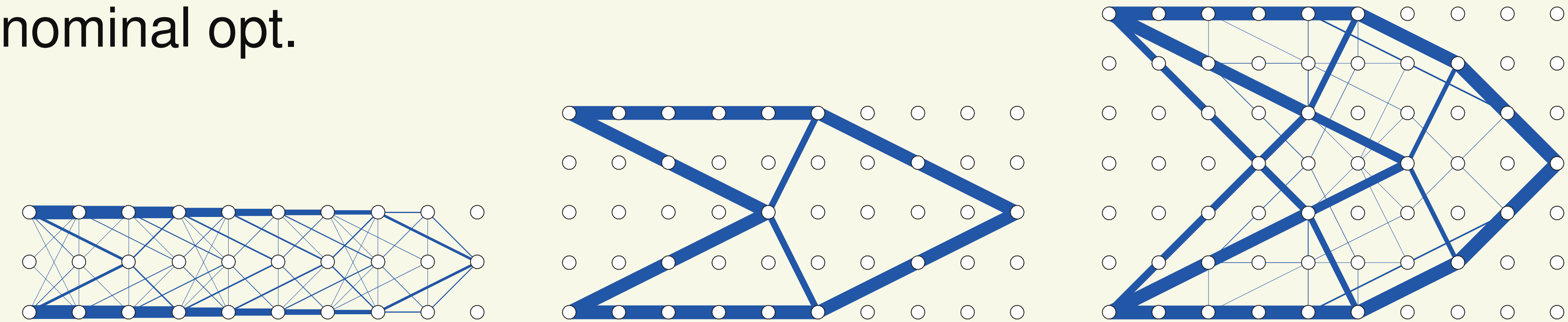
- In robust opt.:
  - Chains are replaced by long single members.
  - (max. mbr. length in sol.) $\leq$ (max. mbr. length in GS) $= 3\,\mathrm{m}$.
  - Less nodes means less uncertain external forces.

num. expt. (3)

- robust opt.



- nominal opt.



(N_X, N_Y)	#memb.	#iter.	Time (s)
(9, 2)	294	28	100.7
(9, 4)	448	24	286.4
(9, 6)	692	35	863.0

conclusions

- robust truss topology optimization against load uncertainty
  - topology-dependent uncertainty
    - uncertain loads at all existing nodes
  - overlapping members
    - should be included in aground structure, but,
    - presence in a solution should be prohibited.
- formulation
  - SDPCC (semidefinite program w/ complementarity constraints)
  - equivalent DC (difference-of-convex) programming
- efficient heuristic
  - concave-convex procedure
    - popular in data science